1 MAGLEV: Magnetic Levitation Plant

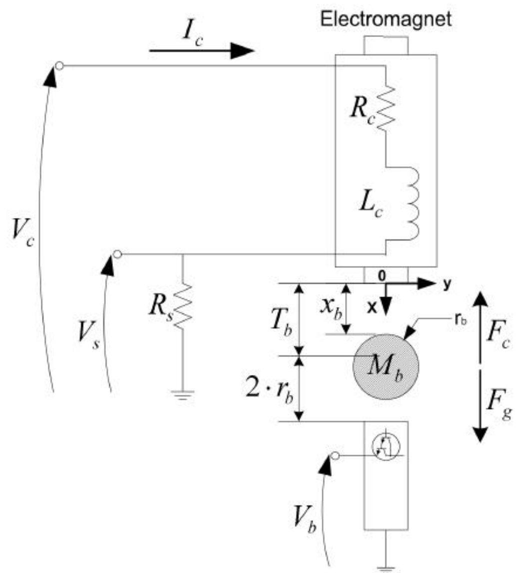
1.1 System Description

The typical Magnetic Levitation plant, i.e. MAGLEV, is depicted in Figure 1, below, while levitating in air a steel ball within its magnetic field. The MAGLEV can be described by three distinct sections encased in a rectangular enclosure. First, the upper section contains an electromagnet, made of a solenoid coil with a steel core. Second, the middle section consists of an inside chamber where the magnetic ball suspension actually takes place. One of the electromagnet poles faces the top of a black post upon which a one-inch steel ball rests. The ball elevation from the post top is measured using a photo-sensitive sensor embedded in the post. The post is designed in such a way that when the ball rests on top of it, the air gap between the ball's top hemisphere and the electromagnet pole face is 14 mm. The post also provides repeatable initial conditions for control system performance evaluation. Finally, the bottom section of the MAGLEV apparatus houses the system's conditioning circuitry needed, for example, by the light intensity position sensor. As detailed later in this manual, both offset and gain potentiometers of the ball position sensor are readily available for proper calibration. A current sense resistor is also included in the design in order to provide for coil current measurement if necessary.

Figure 1 MAGLEV Specialty Plant

1.2 Maglev System Representation

A schematic of the MAGLEV plant is represented in Fig. below. The MAGLEV systems's nomenclature is provided in Section 1.3. The positive direction of vertical displacement is downwards, with the origin of the global Cartesian frame of coordinates on the electromagnet core flat face. Although the ball does have six degrees of freedom, in free space, only the vertical(x) axis is controlled. It can also seen that the MAGLEV consists of two main systems: an Electrical and an electro-mechanical.



Schematic of the MAGLEV Plant

1.2.1 Electrical System Modeling

Derive the mathematical model of MAGLEV electrical system. The resulting model will provide you the open-loop transfer of coil voltage to coil curent unction $G_c(s) = \frac{I_c(s)}{V_c(s)}$

1.2.2 Electro-Mechanical System: Non-Linear Equation of Motion (EOM)

Using the notations and conventions described in Fig. and hints below, derive the Equation of motion (EOM) of MAGLEV electro-mechanical system.
Hint 1: The attractive force, F_c generated by the electromagnet and acting

on the steel ball is assumed to be expressed as

$$F_c = \frac{1}{2}\frac{K_m I_c^2}{x_b^2} \quad for 0 < x_b \tag{1}$$

Equation (1) shows that the pull of the electromagnet is proportional to the square of the current and inversely proportional to the air gap (a.k.a ball position) squared.
Hint 2: The Newton's second law of motion can be applied to the steel ball
Hint 3: Express the resulting EOM under the following format:

$$\frac{\partial^2}{\partial t^2} x_b = f(x_b, I_c) \tag{2}$$

where f denotes a function.

1.3 Maglev Model Parameters

Table below, lists and characterizes the main parameters (e.g. mechanical and electrical specifications, convertion factors, constants) associated with the MAGLEV specialty plant. Some of these parameters can be used for mathematical modelling of the MAGLEV system as well as to obtain the steel ball's Equation Of Motion (EOM).

Symbol	*Description*	*Value*	*Unit*
I_{c_max}	Maximum Continuous Coil Current	3	A
L_c	Coil Inductance	412.5	mH
R_c	Coil Resistance	10	Ω
N_c	Number Of Turns in the Coil Wire	2450	
l_c	Coil Length	0.0825	m
r_c	Coil Steel Core Radius	0.008	m
K_m	Electromagnet Force Constant	6.5308E-005	$N.m^2/A^2$
R_s	Current Sense Resistance	1	Ω
r_b	Steel Ball Radius	1.27E-002	m
M_b	Steel Ball Mass	0.068	kg
T_b	Steel Ball Travel	0.014	m
g	Gravitational Constant on Earth	9.81	m/s^2
μ_0	Magnetic Permeability Constant	4π E-007	H/m
K_B	Ball Position Sensor Sensitivity (Assuming a User-Calibrated Sensor Measurement Range from 0 to 4.95 V)	2.83E-003	m/V

Symbol	***Description***	***Units***	***Matlab Notations***
L_c	Coil Inductance	mH	Lc
R_c	Coil Resistance	Ω	Rc
N_c	Number Of Turns in the Coil Wire		Nc
r_c	Coil Core Radius	m	rc
R_s	Current Sense Resistance	Ω	Rs
r_b	Steel Ball Radius	m	rb
M_b	Steel Ball Mass	kg	Mb
T_b	Steel Ball Travel	m	Tb
g	Gravitational Constant on Earth	m/s^2	g
μ_0	Magnetic Permeability Constant	H/m	mu0
K_B	Ball Position Sensor Sensitivity	m/V	K_B
F_c	Electromagnet Force	N	Fc
F_g	Gravity Force	N	Fg
K_m	Electromagnet Force Constant	$N.m^2/A^2$	Km
I_c	Actual Coil Current	A	Ic
V_c	Actual Coil Input Voltage	V	Vc
V_s	Current Sense Voltage	V	Vs
V_b	Ball Position Sensor Voltage	V	Vb
x_b	Actual Air Gap Between Core Face and Ball Surface (a.k.a. Steel Ball Vertical Position)	m	xb
$\frac{\partial}{\partial t} x_b$	Steel Ball Vertical Velocity	m/s	xb_dot
x_{b0}	Steady-State Air Gap	m	xb0
I_{c0}	Steady-State Coil Current	A	Ic0
x_{b1}	Small Variation Around the Steady-State Air Gap	m	xb1
I_{c1}	Small Variation Around the Steady-State Coil Current	A	Ic1
I_{c_des}	Desired Coil Current	A	Ic_des
x_{b_des}	Desired Air Gap	m	xb_des
I_{c_ff}	Feedforward Coil Current	A	Ic_ff