Nonlinear Control
Lecture 8: Nonlinear Control System Design

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Nonlinear Control Problems
- Stabilization Problems
- Tracking Problems

Specify the Desired Behavior

Some Issues in Nonlinear Control
- Modeling Nonlinear Systems
- Feedback and FeedForward
- Importance of Physical Properties

Available Methods for Nonlinear Control

Nonlinear Control Problems

- **Objective of Control design:** given a physical system to be controlled and specifications of its desired behavior, construct a feedback control law to make the closed-loop system display the desired behavior.
- **Control problems:**
  1. Stabilization (regulation): stabilizing the state of the closed-loop system around an Equ. point, like: temperature control, altitude control of aircraft, position control of robot manipulator.
  2. Tracking (Servo): makes the system output tracks a given time-varying trajectory such as aircraft fly along a specified path, a robot manipulator draw straight lines.
  3. Disturbance rejection or attenuation: rejected undesired signals such as noise
  4. Various combination of the three above

Stabilization Problems

- **Asymptotic Stabilization Problem**: *Given a nonlinear dynamic system:*

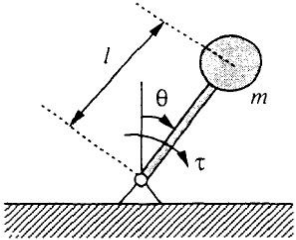
$$\dot{x} = f(x, u, t)$$

find a control law, u, s.t. starting from anywhere in region $\Omega \rightsquigarrow x \longrightarrow 0$ *as* $t \longrightarrow \infty$.

- *If the objective is to drive the state to some nonzero set-point* x_d*, it can be simply transformed into a zero-point regulation problem* $x - x_d$ *as the state.*
  - Static control law: the control law depends on the measurement signal directly, such as proportional controller.
  - Dynamic control law: the control law depends on the measurement through a differential Eq, such as lag-lead. controller

Example: Stabilization of a Pendulum

- Consider the dynamics of the pendulum:
$$J\ddot{\theta} - mgl\sin\theta = \tau$$
- Objective: take the pendulum from a large initial angel ($\theta = 60^o$) to the vertical up position
- A choice of stabilizer:
a feedback part for stability (PD)+
a feedforward part for gravity compensation:
$$\tau = -k_d\dot{\theta} - k_p\theta - mgl\sin\theta$$
k_d and k_p are pos. constants.
- ∴ globally stable closed-lop dynamics:prove it
$$J\ddot{\theta} + k_d\dot{\theta} + k_p\theta = 0$$
- In this example feedback (FB) and feedforward (FF) control actions modify the plant into desirable form.



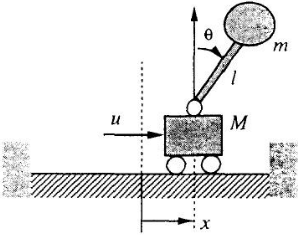
Example: Stabilization of a Inverted Pendulum with Cart

- Consider the dynamics of the inverted pendulum shown in Fig.:

$$(M+m)\ddot{x} + ml\cos\theta\ddot{\theta} - ml\sin\theta\dot{\theta}^2 = u$$

$$m\ddot{x}\cos\theta + ml\ddot{\theta} - ml\dot{x}\dot{\theta}\sin\theta + mg\sin\theta = 0$$

  mass of the cart is not negligible



- Objective: Bring the inverted pendulum from vertical-down at the middle of the lateral track to the vertical-up at the same lateral point.
- It is not simply possible since degree of freedom is two, # inputs is one (under actuated).

Tracking Problems

- **Asymptotic Tracking Problem**: *Given a nonlinear dynamics:*

$$\dot{x} = f(x, u, t)$$
$$y = h(x)$$

and a desired output, y_d, find a control law for the input u s.t. starting form any initial state in region Ω, the tracking error $y(t) - y_d(t)$ goes to zero, while whole state x remain bounded.

- A practical point: Sometimes x can just be remained *reasonably bounded*, i.e., bounded within the range of system model validity.
- Perfect tracking: proper initial states imply zero tracking error for all time: $y(t) \equiv y_d(t) \;\; \forall t \geq 0$ in asymptotic/ exponential tracking perfect tracking is achieved asymptotically/ exponentially
- Assumption throughout the rest of the lectures:
  - y_d and its derivatives up to a sufficiently high order (generally equal to the system's order) are cont. and bounded.
  - y_d and its derivatives available for on-line control computation
  - y_d is planned ahead

- Sometimes derivatives of the desired output are not available.
- A reference model is applied to provide the required derivative signals
- **Example**: For tracking control of the antenna of a radar, only the position of the aircraft $y_a(t)$ is available at a given time instant (it is too noisy to be differentiated numerically).
- desired position, velocity and acceleration to be tracked is obtained by

$$\ddot{y}_d + k_1 \dot{y}_d + k_2 y_d = k_2 y_a(t) \tag{1}$$

  k_1 and k_2 are pos. constants
- $\therefore$ following the aircraft is translated to the problem of tracking the output y_d of the reference model
- The reference model serves as
  - providing the desired output of the tracking system in response to the aircraft position
  - generating the derivatives of the desired output for tracker design.
- (1) Should be fast y_d to closely approximate y_a

Tracking Problem

- Perfect tracking and asymptotic tracking is not achievable for non-minimum phase systems.
- **Example:** Consider $\ddot{y} + 2\dot{y} + 2y = -\dot{u} + u$.
- It is non-minimum phase since it has zero at $s = 1$.
- Assume the perfect tracking is achieved.
- $\therefore\ \dot{u} - u = -(\ddot{y}_d + 2\dot{y}_d + 2y_d) \Rightarrow u = -\frac{s^2+2s+2}{s-1} y_d$
- Perfect tracking is achieved by infinite control input.
- $\therefore$ Only bounded-error tracking with small tracking error is achievable for desired traj.
- Perfect tracking controller is inverting the plant dynamics

Relation between Stabilization and Tracking Problems

- Tracking problems are more difficult to solve than stabilization problems
- In tracking problems the controller should
  - not only keep the whole state stabilized
  - but also drive the system output toward the desired output
- However, for tracking problem of the plant:

$$\ddot{y} + f(\dot{y}, y, u) = 0$$

  - $e(t) = y(t) - y_d(t)$ goes to zero
  - It is equivalent to the asymptotic stabilization of the system

$$\ddot{e}_d + f(\dot{e}, e, u, y_d, \dot{y}_d, \ddot{y}_d) = 0 \tag{2}$$

  with states e and $\dot{e}$

- $\therefore$ tracking problem is solved if we can design a stabilizer for the non-autonomous dynamics (2)
- On the other hand, stabilization problems can be considered as a special case of tracking problem with desired trajectory being a constant.

Specify the Desired Behavior

- In Linear control, the desired behavior is specified in
  - time domain: rise time, overshoot and settling time for responding to a step command
  - frequency domain: the regions in which the loop transfer function must lie at low and high frequencies
- So in linear control the quantitative specifications of the closed-loop system is defined, the a controller is synthesized to meet the specifications
- For nonlinear systems the system specification of nonlinear systems is less obvious since
  - response of the nonlinear system t one command does not reflect the response to an other command
  - a frequency description is not possible
- $\therefore$ In nonlinear control systems some qualitative specifications of the desired behavior is considered.

- Some desired qualitative specifications of nonlinear system:
  - Stability must be guaranteed for the nominal model, either in local or global sense. In local sense, the region of stability and convergence are of interest.
    - stability of nonlinear systems depends on initial conditions and only temporary disturbances may be translated as initial conditions
    - $\therefore$ stability does not imply to withstand persistent disturbance, even in small magnitude
  - Robustness is the sensitivity effect which are not considered in the design like persistent disturbance, measurement noise, unmodeled dynamics, etc.
  - Accuracy and Speed of response for some typical motion trajectories in the region of operation. For instance, sometimes appropriate control is desired to guarantee consistent tracking accuracy independent of the desired traj.
  - Cost of a control which is determined by # and type of actuators, sensors, design complexity.
- The mentioned qualitative specifications are not achievable in a unified design.
- A good control can be obtained based on effective trade-offs of them.

Nonlinear Control Problems

- A Procedure of designing control
  1. Specify the desired behavior and select actuators and sensors
  2. model the physical plant by a set of differential Eqs
  3. design a control law
  4. analyze and simulate the resulting control system
  5. implement the control system in hardware
- Experience and creativity of important factor in designing the control
- Sometimes, addition or relocation of actuators and sensors may make control of the system easier.
- **Modeling Nonlinear Systems**
- Modeling is constructing a mathematical description (usually as a set of differential Eqs.) for the physical system to be controlled.

Modeling Nonlinear Systems

- Two points in modeling:
  1. To obtain tractable yet accurate model, good understanding of system dynamics and control tasks requires.
     - Note: more accurate models are not always better. They may require unnecessarily complex control design and more computations.
     - Keep essential effects and discard insignificant effects in operating range of interest.
  2. In modeling not only the nominal model for the physical system should be obtained, but also some characterization of the model uncertainties should be provided for using in robust control, adaptive design or simulation.
- Model uncertainties: difference between the model and real physical system
  - parametric uncertainties: uncertainties in parameters
- **Example:** model of controlled mass: $m\ddot{x} = u$
  - Uncertainty in m is parametric uncertainty
  - neglected motor dynamics, measurement noise, and sensor dynamics are non-parametric uncertainties.
  - Parametric uncertainties are easier to characterize; $2 \leq m \leq 5$

Feedback and FeedForward

- Feedback (FB) plays a fundamental role in stabilizing the linear as well as nonlinear control systems
- Feedforward (FF) in nonlinear control is much more important than linear control
- FF is used to
  - cancel the effect of known disturbances
  - provide anticipate actions in tracking tasks
- for FF a model of the plant (even not very accurate) is required.
- Many tracking controllers can be written in the form: $u = \text{FF} + \text{FB}$
  - FF: to provide necessary input to follow the specified motion traj and canceling the effect of known disturbances
  - FB to stabilize the tracking error dynamics.

Example

- Consider a minimum-phase system

$$A(s)y = B(s)u \tag{3}$$

where $A(s) = a_0 + a_1 s + \ldots + a_{n-1}s^{n-1} + s^n, \; B(s) = b_0 + b_1 s + \ldots + b_m s^m$

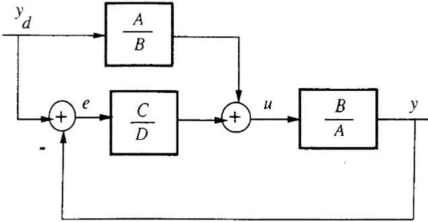
- Objective: make the output $y(t)$ follow a time-varying traj $y_d(t)$
  1. To achieve $y = y_d$, input should have a FF term of $\frac{A(s)}{B(s)}$:

$$u = v + \frac{A(s)}{B(s)} y_d \tag{4}$$

     - Substitute (4) to (3): $A(s)e = B(s)v$, where $e(t) = y(t) - y_d(t)$
  2. Use FB to stabilize the system:
     - $v = \frac{C(s)}{D(s)} \rightsquigarrow$ closed loop system $(AC + BD)e = 0$.
     - Choose D and C to poles in desired places
- $\therefore u = \frac{A}{B} y_d + \frac{C}{D} e$
- $e(t)$ is zero if initial conditions $y^{(i)}(0) = y_d^{(i)}(0), \; i = 1, \ldots, r$,otherwise exponentially converges to zero

Example Cont'd

- If some derivatives of y_d are not available, one can simply omit them from FF $\rightsquigarrow$ only bounded tracking error is guaranteed,
- The mentioned method is not applicable for non-minimum phase systems.
  - By FF low frequency components of desired traj, good tracking in low freq (lower than the LHP zeros of plant) may be still achieved
  - By defining FF term as $\frac{A}{B_1} y_d \rightsquigarrow e(t) = \frac{D}{AD-BC}[\frac{B}{B_1} - 1]Ay_d$
  - If B_1 is close to B provides good tracking with freq lower than the RHP zeros.



Linear Tracking Control system

Importance of Physical Properties

- In nonlinear control design, explanation of the physical properties may make the control of complex nonlinear plants simple;
- **Example:** Adaptive control of robot manipulator was long recognized to be far of reach.
- Because robot's dynamics is highly nonlinear and has multiple inputs
- Using the two physical facts:
  - pos. def. of inertia matrix
  - possibility if linearly parameterizing robot dynamics

  yields adaptive control with global stability and desirable tacking convergence.

Available Methods for Nonlinear Control

- There is no general method for designing nonlinear control
- Some alternative and complementary techniques to particular classed of control problem are listed below:
  - **Trail-and Error**: The idea is using analysis tools such a phase-plane methods, Lyapunov analysis , etc, to guide searching a controller which can be justified by analysis and simulations.
    - This method fails for complex systems
  - **Feedback Liberalization:** transforms original system models into equivalent models of simpler form (like fully or partially linear)
    - Then a powerful linear design technique completes the control design
    - This method is applicable for input-state linearizable and minimum phase systems
    - It requires full state measurement
    - It does not guarantee robustness in presence of parameter uncertainties or disturbances.
    - It can be used as model-simplifying for robust or adaptive controllers

Available Methods for Nonlinear Control

- **Robust Control** is deigned based on consideration of nominal model as well as some characterization of the model uncertainties
  - An example of robust controls is sliding mode control
  - They generally require state measurements.
  - In robust control design tries to meet the control objective for any model in the "ball of uncertainty."
- **Adaptive Control** deals with uncertain systems or time-varying systems.
  - They are mainly applied for systems with known dynamics but unknown constant or slowly-varying parameters.
  - They parameterizes the uncertainty in terms of certain unknown parameters and use feedback to learn these parameters on-line , during the operation of the system.
  - In a more elaborate adaptive scheme, the controller might be learning certain unknown nonlinear functions, rather than just learning some unknown parameters.