Neural Networks
Lecture 8: Identification Using Neural Networks

H.A Talebi
Farzaneh Abdollahi

Department of Electrical Engineering

Amirkabir University of Technology

- Engineers desired to model the systems by mathematical models.
- This model can expressed by operator f from input space u into an output space y.
- **System Identification problem:** is finding $\hat{f}$ which approximates f in desired sense.
  - Identification of static systems: A typical example is pattern recognition:
    - Compact sets $u_i \in \mathcal{R}^n$ are mapped into elements $y_i \in \mathcal{R}^m$ in the output
  - Identification of dynamic systems: The operator f is implicitly defined by I/O pairs of time function $u(t), y(t), t \in [0, T]$ or in discrete time:

$$y(k+1) = f(y(k), y(k-1), ..., y(k-n), u(k), ..., u(k-m)), \tag{1}$$

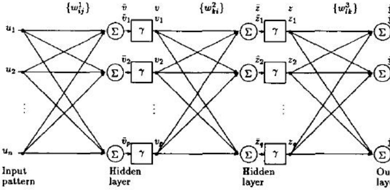
- In both cases the objective to determine $\hat{f}$ is

$$\|\hat{y} - y\| = \|\hat{f} - f\| \leq \epsilon, \text{for some desired} \epsilon > 0.$$

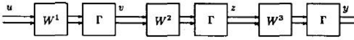
- Behavior of systems in practice are mostly described by dynamical models.
- $\therefore$ Identification of dynamical systems is desired in this lecture.
- In identification problem, it is always assumed that the system is stable

Representation of Dynamical Systems by Neural Networks

1. Using Static Networks: Providing the dynamics out of the network and apply static networks such as multilayer networks (MLN).
   - Consists of an input layer, output layer and at least one hidden layer
   - In fig. there are two hidden layers with three weight matrices W_1, W_2 and W_3 and a diagonal nonlinear operator Γ with activation function elements.
   - Each layer of the network can be represented by $N_i[u] = \Gamma[W_i u]$.
   - The I/O mapping of MLN can be represented by $y = N[u] = \Gamma[W_3\Gamma[W_2\Gamma[W_1 u]]] = N_3 N_2 N_1[u]$
   - The weights W_i are adjusted s.t min a function of the error between the network output y and desired output y_d.



A three layer neural network.



A block diagram representation of a three layer ne

Using Static Networks

- The *universal approximation theorem* shows that a three layers NN with a backpropagation training algorithm has the potential of behaving as a universal approximator
- **Universal Approximation Theorem:** *Given any* $\epsilon > 0$ *and any* $\mathcal{L}_2$ *function* $f : [0,1]^n \in \mathcal{R}^n \rightarrow \mathcal{R}^m$, *there exists a three-layer backpropagation network that can approximate* f *within* ϵ *mean-square error accuracy.*

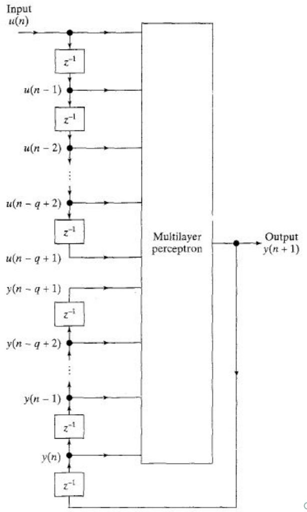
Using Static Networks

- Providing dynamical terms to inject to static networks:
  1. Tapped-Delay-Lines (TDL): Consider (1) for identification (I/O Model)

$$y(k+1) = f(y(k), y(k-1), \dots, y(k-n), u(k), \dots, u(k-m)),$$

  - Dynamical terms $u(k-j), y(k-i)$ for $i = 1, \dots, n, j = 1, \dots, m$ is made by delay elements out of the network and injected to the network as input.
  - The static network is employed to approximate the function f
  - $\therefore$ The model provided by the network will be

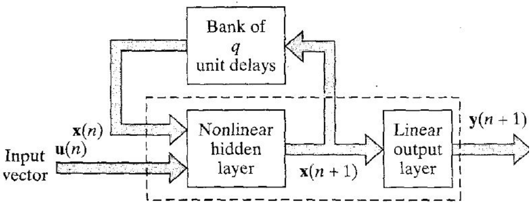
$$\hat{y}(k+1) = \hat{f}(\hat{y}(k), \hat{y}(k-1), \dots, \hat{y}(k-n), u(k), \dots, u(k-m)),$$



Using Static Networks

- Considering State Space model:

$$x(k+1) = f(x(k), x(k-1), ..., x(k-n), u(k), ..., u(k-m)),$$
$$y(k) = Cx(k)$$



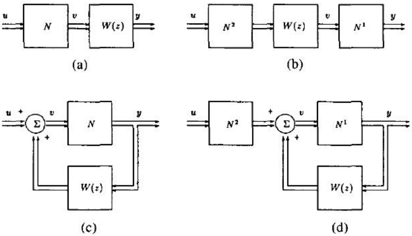
Using Static Networks

2 Filtering

- in continuous-time networks the delay operator can be shown by integrator.
- The dynamical model can be represented by an MLN , $N_1[.]$, + a transfer matrix of linear function, $W(s)$.
- For example:

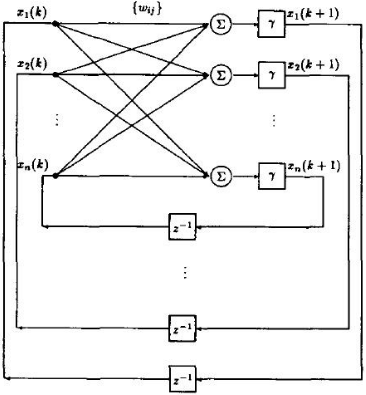
$$\dot{x}(t) = f(x,u) \pm Ax,$$

- where A is Hurwitz. Define $g(x,u) = f(x,u) - Ax$
- $\dot{x} = g(x,u) + Ax$
- Fig, shows 4 configurations using filter.



Representation of Dynamical Systems by Neural Networks

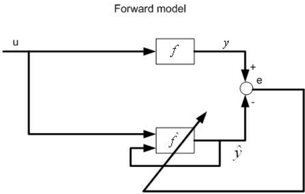
2. Using Dynamic Networks: Time-Delay Neural Networks (TDNN) [?] , Recurrent networks such as Hopfield:
   - Consists of a single layer network N_1, included in feedback configuration and a time delay
   - Can represent discrete-time dynamical system as :
     $x(k+1) = N_1[x(k)], \quad x(0) = x_0$
   - If N_1 is suitably chosen, the solution of the NN converge to the same equilibrium point of the system.
   - In continuous-time, the feedback path has a diagonal transfer matrix with $1/(s-\alpha)$ in diagonal.
   - $\therefore$ the system is represented by
     $\dot{x} = \alpha x + N_1[x] + I$



The Hopfield network.

Neural Networks Identification Model

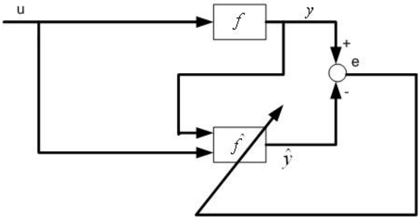
- Two principles of identification problems:
  1. Identification model
  2. Method of adjusting its parameters based on identification error $e(t)$
- **Identification Model**
  1. Direct modeling:
     - it is applicable for control, monitoring, simulation, signal processing
     - The objective: output of NN $\hat{y}$ converge to output of the system $y(k)$
     - $\therefore$ the signal of target is output of the system
     - Identification error $e = y(k) - \hat{y}(k)$ can be used for training.
     - The NN can be a MLN training with BP, such that minimizes the identification error.
     - The structure of identification shown in Fig named **Parallel Model**



Direct Modeling

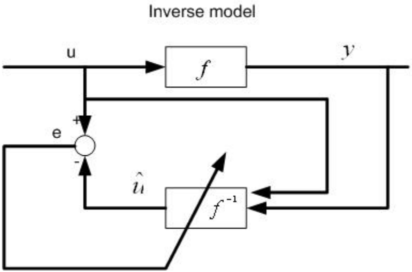
- Drawback of parallel model: There is a feedback in this model which some times makes convergence difficult or even impossible.

2. **Series-Parallel Model**
    - In this model the output of system is fed to the NN



Inverse Modeling

- It is employed for the control techniques which require inverse dynamic
- Objective is finding f^{-1}, i.e., $y \rightarrow u$
- Input of the plant is target, u
- Error identification is defined $e = u - \hat{u}$



Example 1: Using Filtering

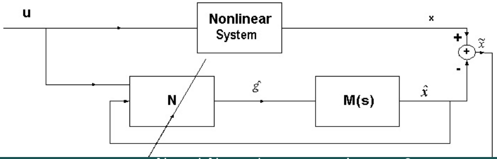
- Consider the nonlinear system

$$\dot{x} = f(x,u) \quad (2)$$

  - $u \in R^m$: input vector, $x \in R^n$: state vector, $f(.)$: an **unknown** function.
  - Open loop system is stable.
  - Objective: Identifying f
- Define filter:
  - Adding Ax to and subtracting from (2), where A is an **arbitrary** Hurwitz matrix

$$\dot{x} = Ax + g(x,u) \quad (3)$$

    where $g(x,u) = f(x,u) - Ax$.
- Corresponding to the Hurwitz matrix A, $M(s) := (sI - A)^{-1}$ is an $n \times n$ matrix whose elements are stable transfer functions.



- The model for identification purposes:

$$\dot{\hat{x}} = A\hat{x} + \hat{g}(\hat{x}, u)$$

- The identification scheme is based on the *parallel* configuration
  - The states of the model are fed to the input of the neural network.
  - an MLP with at least three layers can represent the nonlinear function g as:

$$g(x, u) = W\sigma(V\bar{x})$$

- W and V are the ideal but **unknown** weight matrices
- $\bar{x} = [x \quad u]^T$,
- $\sigma(.)$ is the transfer function of the hidden neurons that is usually considered as a sigmoidal function:

$$\sigma_i(V_i\bar{x}) = \frac{2}{1 + exp^{-2V_i\bar{x}}} - 1$$

- where V_i is the *ith* row of V,
- $\sigma_i(V_i\bar{x})$ is the *ith* element of $\sigma(V\bar{x})$.

- g can be approximated by NN as

$$\hat{g}(\hat{x}, u) = \hat{W}\sigma(\hat{V}\hat{\bar{x}})$$

- The identifier is then given by

$$\dot{\hat{x}}(t) = A\hat{x} + \hat{W}\sigma(\hat{V}\hat{\bar{x}}) + \epsilon(x)$$

- $\epsilon(x) \leq \epsilon_N$ is the neural network's bounded approximation error
- the error dynamics:

$$\dot{\tilde{x}}(t) = A\tilde{x} + \tilde{W}\sigma(\hat{V}\hat{\bar{x}}) + w(t)$$

  - $\tilde{x} = x - \hat{x}$: identification error
  - $\tilde{W} = W - \hat{W}$, $w(t) = W[\sigma(V\bar{x}) - \sigma(\hat{V}\hat{\bar{x}})] - \epsilon(x)$ is a bounded disturbance term, i.e, $\|w(t)\| \leq \bar{w}$ for some pos. const. $\bar{w}$, due to the sigmoidal function.

- Objective function $J = \frac{1}{2}(\tilde{x}^T\tilde{x})$

- Training:
  - Updating weights:

$$\dot{\hat{W}} = -\eta_1\left(\frac{\partial J}{\partial \hat{W}}\right) - \rho_1\|\tilde{x}\|\hat{W}$$
$$\dot{\hat{V}} = -\eta_2\left(\frac{\partial J}{\partial \hat{V}}\right) - \rho_2\|\tilde{x}\|\hat{V}$$

  - Therefore:

$$net_{\hat{v}} = \hat{V}\hat{\bar{x}}$$
$$net_{\hat{w}} = \hat{W}\sigma(\hat{V}\hat{\bar{x}}).$$

  - $\frac{\partial J}{\partial \hat{W}}$ and $\frac{\partial J}{\partial \hat{V}}$ can be computed according to

$$\frac{\partial J}{\partial \hat{W}} = \frac{\partial J}{\partial net_{\hat{w}}} \cdot \frac{\partial net_{\hat{w}}}{\partial \hat{W}}$$
$$\frac{\partial J}{\partial \hat{V}} = \frac{\partial J}{\partial net_{\hat{v}}} \cdot \frac{\partial net_{\hat{v}}}{\partial \hat{V}}$$

$$\frac{\partial J}{\partial net_{\hat{w}}} = \frac{\partial J}{\partial \tilde{x}} \cdot \frac{\partial \tilde{x}}{\partial \hat{x}} \cdot \frac{\partial \hat{x}}{\partial net_{\hat{w}}} = -\tilde{x}^T \cdot \frac{\partial \hat{x}}{\partial net_{\hat{w}}}$$
$$\frac{\partial J}{\partial net_{\hat{v}}} = \frac{\partial J}{\partial \tilde{x}} \cdot \frac{\partial \tilde{x}}{\partial \hat{x}} \cdot \frac{\partial \hat{x}}{\partial net_{\hat{v}}} = -\tilde{x}^T \cdot \frac{\partial \hat{x}}{\partial net_{\hat{v}}}$$

- and

$$\frac{\partial net_{\hat{w}}}{\partial \hat{W}} = \sigma(\hat{V}\hat{\hat{x}})$$
$$\frac{\partial net_{\hat{v}}}{\partial \hat{V}} = \hat{\hat{x}}$$
$$\frac{\partial \dot{\hat{x}}(t)}{\partial net_{\hat{w}}} = A\frac{\partial \hat{x}}{\partial net_{\hat{w}}} + \frac{\partial \hat{g}}{\partial net_{\hat{w}}}$$
$$\frac{\partial \dot{\hat{x}}(t)}{\partial net_{\hat{v}}} = A\frac{\partial \hat{x}}{\partial net_{\hat{v}}} + \frac{\partial \hat{g}}{\partial net_{\hat{v}}}.$$

- Which is dynamic BP. Modify BP algorithm s.t. the static approximations of $\frac{\partial \hat{x}}{\partial net_{\hat{w}}}$ and $\frac{\partial \hat{x}}{\partial net_{\hat{v}}}$ $(\dot{\hat{x}} = 0)$

- Thus,

$$\frac{\partial \hat{x}}{\partial net_{\hat{w}}} = -A^{-1}$$
$$\frac{\partial \hat{x}}{\partial net_{\hat{v}}} = -A^{-1}\hat{W}(I - \Lambda(\hat{V}\hat{\bar{x}}))$$

where

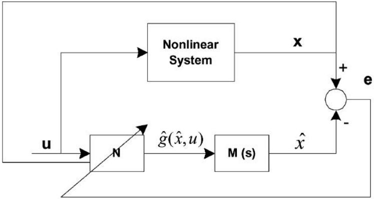
$$\Lambda(\hat{V}\hat{\bar{x}}) = diag\{\sigma_i^2(\hat{V}_i\hat{\bar{x}})\}, i = 1, 2, ..., m.$$

- Finally

$$\begin{aligned} \dot{\hat{W}} &= -\eta_1(\tilde{x}^T A^{-1})^T(\sigma(\hat{V}\hat{\bar{x}}))^T \\ &- \rho_1\|\tilde{x}\|\hat{W} \\ \dot{\hat{V}} &= -\eta_2(\tilde{x}^T A^{-1}\hat{W}(I - \Lambda(\hat{V}\hat{\bar{x}})))^T\hat{\bar{x}}^T \\ &- \rho_2\|\tilde{x}\|\hat{V} \end{aligned}$$

- $\tilde{W} = W - \hat{W}$ and $\tilde{V} = V - \hat{V}$,
- It can be shown that $\tilde{x}$, $\tilde{W}$, and $\tilde{V} \in L_\infty$
- The estimation error and the weights error are all ultimately bounded [?].

- **Series-Parallel Identifier**
  - The function g can be approximated by$\hat{g}(x,u) = \hat{W}\sigma(\hat{V}\bar{x})$
  - Only $\hat{\bar{x}}$ is changed to $\bar{x}$.
  - The error dynamics $\dot{\tilde{x}}(t) = A\tilde{x} + \tilde{W}\sigma(\hat{V}\bar{x}) + w(t)$ where $w(t) = W[\sigma(V\bar{x}) - \sigma(\hat{V}\bar{x})] + \epsilon(x)$
  - only definition of $w(t)$ is changed.
  - Applying this change, the rest remains the same



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Using Dynamic BP Without Static Approximation

$$\frac{\partial J}{\partial \hat{W}} = \frac{\partial J}{\partial \tilde{x}}.\frac{\partial \tilde{x}}{\partial \hat{x}}.\frac{\partial \hat{x}}{\partial net_{\hat{w}}}.\frac{\partial net_{\hat{w}}}{\partial \hat{W}} = -\tilde{x}^T.\frac{\partial \hat{x}}{\partial net_{\hat{w}}}.\sigma(\hat{V}\hat{\bar{x}})$$

$$\frac{\partial J}{\partial \hat{V}} = \frac{\partial J}{\partial \tilde{x}}.\frac{\partial \tilde{x}}{\partial \hat{x}}.\frac{\partial \hat{x}}{\partial net_{\hat{v}}}.\frac{\partial net_{\hat{v}}}{\partial \hat{V}} = -\tilde{x}^T.\frac{\partial \hat{x}}{\partial net_{\hat{v}}}\hat{\bar{x}}$$

- and

$$d_w = \frac{\partial \hat{x}(t)}{\partial net_{\hat{w}}}$$

$$\dot{d}_w = \frac{\partial \dot{\hat{x}}(t)}{\partial net_{\hat{w}}} = Ad_w + \frac{\partial \hat{g}}{\partial net_{\hat{w}}} = Ad_w + 1 \quad (4)$$

$$d_v = \frac{\partial \hat{x}(t)}{\partial net_{\hat{v}}}$$

$$\dot{d}_v = \frac{\partial \dot{\hat{x}}(t)}{\partial net_{\hat{v}}} = Ad_v + \frac{\partial \hat{g}}{\partial net_{\hat{v}}} = Ad_v + \hat{W}(I - \Lambda(\hat{V}\hat{\bar{x}})) \quad (5)$$

Using Dynamic BP Without Static Approximation

- Finally

$$
\begin{aligned}
\dot{\hat{W}} &= \eta_1(\tilde{x}^T d_w)^T(\sigma(\hat{V}\hat{\hat{x}}))^T - \rho_1\|\tilde{x}\|\hat{W} \\
\dot{\hat{V}} &= \eta_2(\tilde{x}^T d_v))T\hat{\hat{x}}^T - \rho_2\|\tilde{x}\|\hat{V}
\end{aligned}
$$

- In learning rule procedure, first (4) and (5) should be solved then the weights W and V is updated

Case Study: Simulation Results on SSRMS

- The Space Station Remote Manipulator System (SSRMS) is a 7 DoF robot which has 7 revolute joints and two long flexible links (booms).
- The SSRMS have no uniform mass and stiffness distributions. Most of its masses are concentrated at the joints, and the joint structural flexibilities contribute a major portion of the overall arm flexibility.
- Dynamics of a flexible–link manipulator

$$M(q)\ddot{q} + h(q,\dot{q}) + Kq + F\dot{q} = u$$

  - $u = [\tau^T \; 0_{1\times m}]^T$, $q = [\theta^T \; \delta^T]^T$,
  - θ is the $n \times 1$ vector of joint variables
  - δ is the $m \times 1$ vector of deflection variables
  - $h = [h_1(q,\dot{q}) \; h_2(q,\dot{q})]^T$: including gravity, Coriolis, and centrifugal forces;
  - M is the mass matrix,
  - $K = \begin{bmatrix} 0_{n\times n} & 0_{n\times m} \\ 0_{m\times n} & K_{m\times m} \end{bmatrix}$ is the stiffness matrix,
  - $F = diag\{F_1, F_2\}$: the viscous friction at the hub and in the structure,
  - τ: input torque.

Case Study: Simulation Results on SSRMS

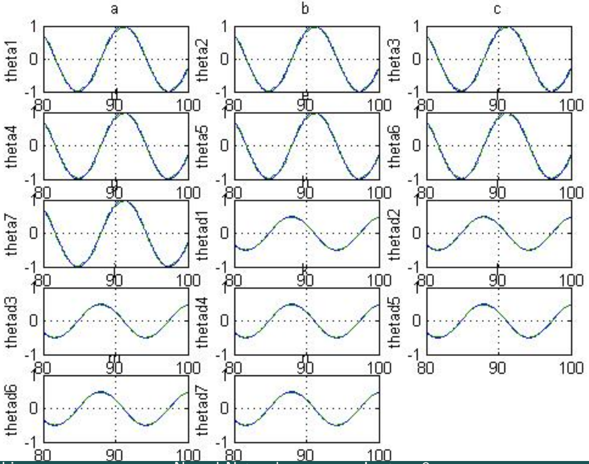
http://english.sohu.com/20050729/n226492517.shtml

Case Study: Simulation Results on SSRMS

- A joint PD control is applied to stabilize the closed-loop system $\rightsquigarrow$ boundedness of the signal $x(t)$ is assured.
- For a two link flexible manipulator
  - $x = [\theta_1 \ldots \theta_7 \; \dot{\theta}_1 \ldots \dot{\theta}_7 \; \delta_{11} \; \delta_{12} \; \delta_{21} \; \delta_{22} \; \dot{\delta}_{11} \; \dot{\delta}_{12} \; \dot{\delta}_{21} \; \dot{\delta}_{22}]^T$
  - The input: $u = [\tau_1, \; \ldots, \; \tau_7]$
  - A is defined as $A = -2I \in \mathcal{R}^{22 \times 22}$
  - Reference trajectory: $sin(t)$
- The identifier:
  - Series-parallel
  - A three-layer NN network: 29 neurons in the input layer, 20 neurons in the hidden layer, and 22 neurons in the output layer.
  - The 22 outputs correspond to
    - 7 joint positions
    - 7 joint velocities
    - 4 in-plane deflection variables
    - 4 out-of plane deflection variables
  - The learning rates and damping factors: $\eta_1 = \eta_2 = 0.1, \; \rho_1 = \rho_2 = 0.001.$

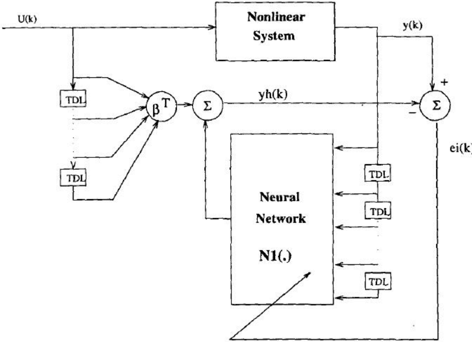
Case Study: Simulation Results on SSRMS

- Simulation results for the SSRMS: (a-g) The joint positions, and (h-n) the joint velocities.



Example 2: TDL

- Consider the following nonlinear system
  $$y(k) = f(y(k-1), ..., y(k-n)) + b_0 u(k) + ... + b_m u(k-m)$$
  - u: input, y:output, $f(.)$: an **unknown** function.
  - Open loop system is stable.
  - Objective: Identifying f
- Series-parallel identifier is applied.
- $\beta = [b_0,\ b_1, ..., b_m]$
- Cost function: $J = \frac{1}{2}e_i^2$ where $e_i = y - y_h$,



- Consider Linear in parameter MLP,
  - In sigmoidal function.σ, the weights of first layer is fixed $V = I$: $\sigma_i(\bar{x}) = \frac{2}{1+exp^{-2\bar{x}}} - 1$
- Updating law: $\triangle w = -\eta(\frac{\partial J}{\partial w})$
- $\therefore \frac{\partial J}{\partial w} = \frac{\partial J}{\partial e_i}\frac{\partial e_i}{\partial w} = -e_i\frac{\partial N(.)}{\partial w}$
- $\frac{\partial N(.)}{\partial w}$ is obtained by BP method.
- Numerical Example: Consider a second order system

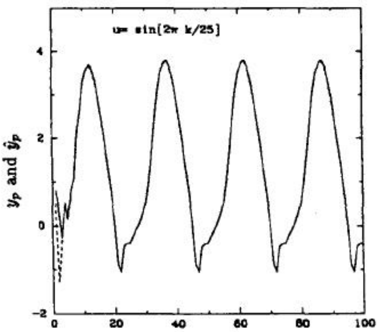
$$y_p(k+1) = f[y_p(k), y_p(k-1)] + u(k)$$

where $f[y_p(k), y_p(k-1)] = \frac{y_p(k)y_p(k-1)[y_p(k)+2.5]}{1+y_p^2(k)+y_p^2(k-1)}$.

- After checking the stability system
- Apply series-parallel identifier
- u is random signal informally is distributed in $[-2,\ 2]$
- $\eta = 0.25$

Numerical Example Cont'd

- The outputs of the plant and the model after the identification procedure



Outputs of the plant and the identification model.

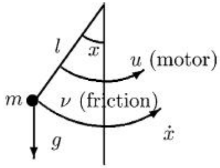
Example 3 [?]

- A gray box identification,(the system model is known but it includes some unknown, uncertain and/or time-varying parameters) is proposed using Hopfield networks
- Consider

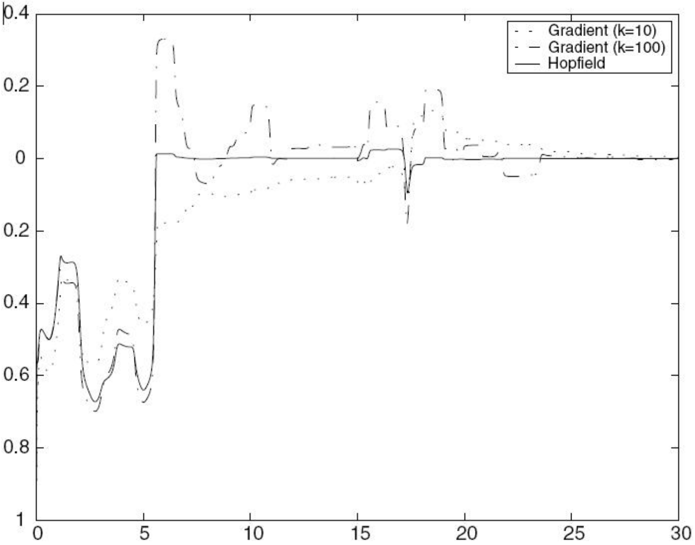
$$\dot{x} = A(x, u(t))(\theta_n + \theta(t))$$
$$y = x$$

  - y is the output,
  - θ is the unknowntime-dependantdeviation from the nominal values
  - A is a matrix that depends on the input u and the state x
  - y and A are assumed to be physically measurable.
- Objective: estimating θ (i.e. min the estimation error: $\tilde{\theta} = \theta - \hat{\theta}$).

- At each time interval assume time is frozen so that $A_c = A(x(t), u(t)),\ y_c = y(t)$
- Recall Gradient-Type Hopefield $C\frac{du}{dt} = Wv(t) + I$
- the weight matrix and the bias vector are defined: $W = -A_c^T A_c,\ I = A_c^T A_c \theta_n - A_c^T y_c$
- The convergence of the identifier is proven using Lyapunov method
- It is examined for an idealized single link manipulator $\ddot{x} = -\frac{g}{l} sinx - \frac{v}{ml^2}\dot{x} + \frac{1}{ml^2}u$
- assume $A = (sinx, \dot{x}, u)$ and $\theta_n + \theta = (-\frac{g}{l},\ \ -\frac{v}{ml^2},\ \ \frac{1}{ml^2})$



Single link manipulator.



Estimation error for θ_1.

References

K.S. Narendra and K. Parthasarathy, "Identification and control of dynamical systems using neural networks," *IEEE Trans. on Neural Networks*, vol. 1, no. 1, pp. 4–27, March 1990.