Signals and Systems
Lecture 6: Fourier Applications

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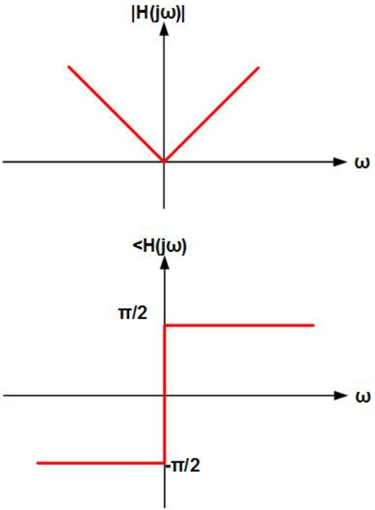
Filtering

- Filtering: is a process to
  - Change the relative frequency components
  - Eliminate some frequency components
- The filters can be
  - Frequency shape filter: changes the shape of spectrum
  - Frequency selective filter: passes some frequencies and significantly attenuate or eliminate others
- For LTI systems, we have: $Y(j\omega) = H(j\omega)X(j\omega)$
- Therefore for LTI systems filtering is defining proper frequency response $H(j\omega)$

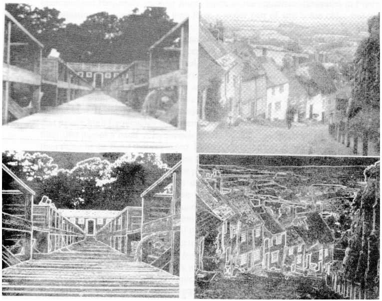
Frequency Shaping Filters

- Example: In audio systems such filters allow the listener to change the relative amount of low frequency energy (bass) and high frequency energy (treble).
- Example: a differentiator:
  $y(t) = \frac{dx}{dt} \rightsquigarrow Y(j\omega) = j\omega X(j\omega)$
  - $H(j\omega) = j\omega$
  - The larger ω the more amplification will receive
  - In control engineering such filter are employed for improving transient response or changing rapid variations (Proportional Differentiators (PD))



Example Cont'd

- Another application of differentiator filters is to enhance edges in picture processing
  - A black-wight picture has a two dimensional signal in x and y directions
  - It requires a two dimensional Fourier series
  - Abrupt changes of brightness across edges leads to more concentration at higher frequency
  - Passing the signal from differentiator enhance this concentration and make it more clear

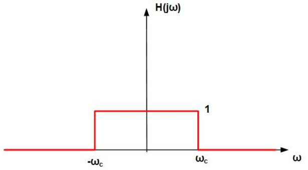


Frequency Selective Filters

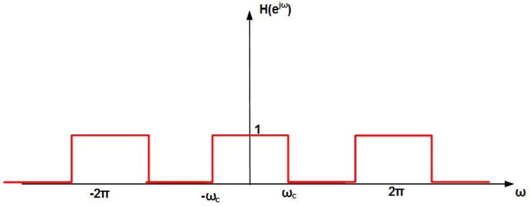
- This filters keep some band of frequencies and eliminate others.
- Example: If there is a noise in an audio recording in high frequency band, it can be removed by such filters
- Example: In Amplitude Modulation (AM), information is transmitted from different sources simultaneously s.t. each channel puts its information in separate frequency band. At receiver (in home radio/TV)
  - Frequency Selective Filters separate the individual channels
  - Frequency Shaping Filters (like equalizer) adjust the quality of tone
- Based on the bound that the Frequency Selective Filters pass, these filters can be categorized to
  - Lowpass filters
  - Highpass filters
  - Bandpass filters

Lowpass Filters

- They passes the lower freqs. (freq. around $\omega = 0$) and attenuate or reject higher freqs.
  - In CT

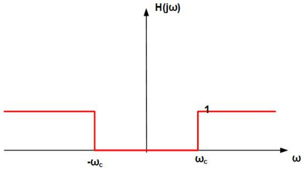


  - In DT (Low freq. is at $\omega = 2k\pi, \quad k = 0, \pm1, \pm2, \ldots$)

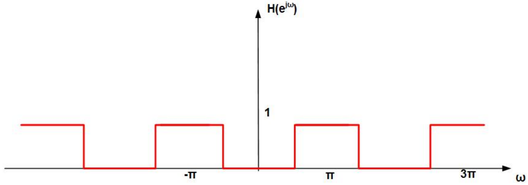


Highpass Filters

- They passes the high freqs. and attenuate or reject low freqs.
  - In CT

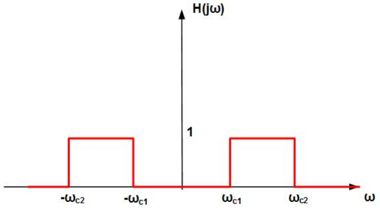


  - In DT (the highest freq. at DT is at $\omega = (k+1)\pi, \quad k = 0, \pm 1, \pm 2, \ldots$ since $e^{j\pi n} = (-1)^n$)

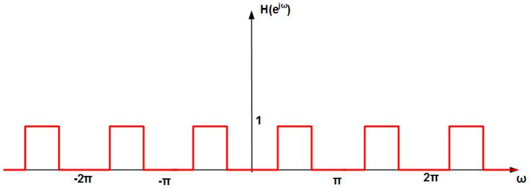


Bandpass Filters

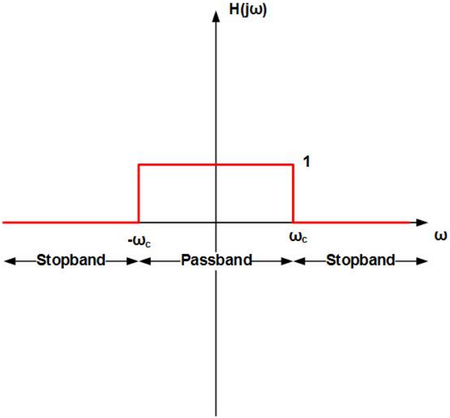
- They passes a band of freqs. and attenuate or reject low and high freqs.
  - In CT



  - In DT



- Pass Band : The band of freq. that is passed through filter
- Stop Band: The band of freq that is rejected by filter
- Cutoff Freq.: The border between pass band and stop band

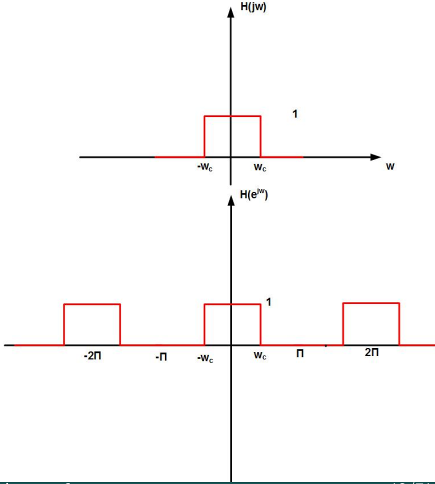


Ideal Filters

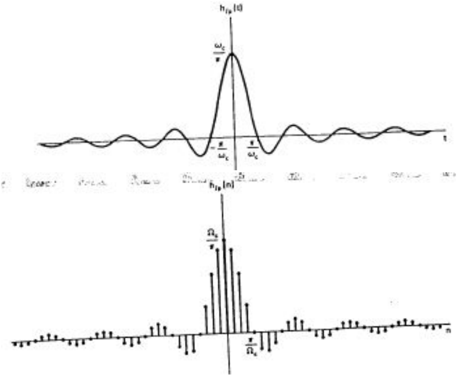
- An ideal filter completely eliminates all signals info at stop band while passing those in pass band unchanged:
- An ideal lowpass filter:
  - In pass band : $|H(j\omega)| = 1, \measuredangle H(j\omega) = 0$
  - In stop band $|H(j\omega)| = 0$
  - It is symmetric around $\omega = 0$
- An ideal low-pass filter can be realized mathematically by multiplying a signal by the rectangular function in the frequency domain
  - $H(j\omega) = \begin{cases} 1 & |\omega| \leq \omega_c \\ 0 & |\omega| > \omega_c \end{cases}$
  - or, convolution with sinc function (its impulse response), in the time domain
- Ideal filters are good for system analysis
- But they are not realizable and implementable in practice

Ideal Filter, Example:

- Consider an ideal low pass filter:
- CT filter: $H(j\omega) = \begin{cases} 1 & |\omega| \leq \omega_c \\ 0 & |\omega| > \omega_c \end{cases} \Leftrightarrow h(t) = \frac{sin\omega_c t}{\pi t}$
- DT filter: $H(e^{j\omega}) = \begin{cases} 1 & |\omega| \leq \omega_c \\ 0 & \omega_c < |\omega| \leq \pi \end{cases} \Leftrightarrow$ $h[n] = \frac{sin\omega_c n}{\pi n}$



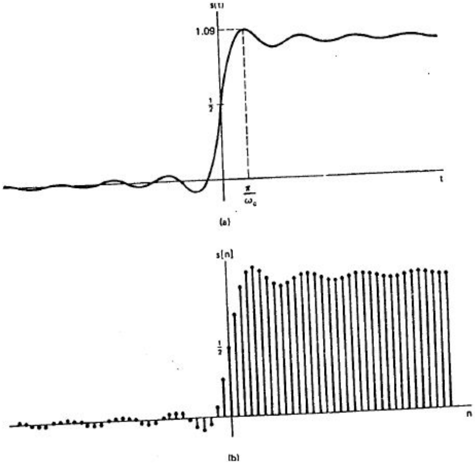
Ideal Filter, Example:



Example Cont'd

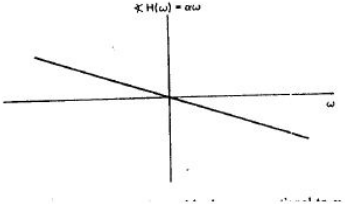
- Lowpass filter is not casual ($h(t)/h[n]$ is not zero for $t < 0/n < 0$)
- Therefore it is not implementable in real
- Moreover in some applications like suspension system oscillating behavior of impulse response of the filter is not desirable.
- In freq. the width of pass band is proportional to ω_c; in time, the width of the main lobe is proportional to $\frac{1}{\omega_c}$
  - To expand pass band in freq. impulse response of the filter should be narrower
- Now let us study the step response of these filters
- Reconsider $s(t) = \int_{-\infty}^{t} h(\tau)d\tau / s[n] = \sum_{-\infty}^{n} h[m]$
- The step responses have overshoot comparing to the final value and they have oscillating response, none of them are desirable

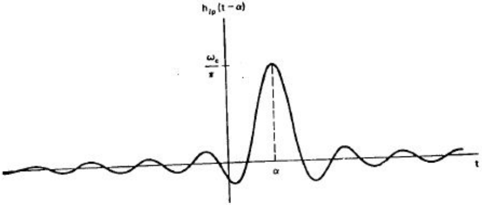
Ideal Filter, Example:



Phase shifting in Ideal Filters

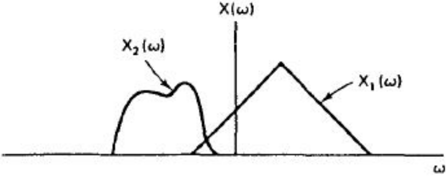
- An ideal filter with linear phase (in pass band) results in a simple time shifting the filter in time domain.



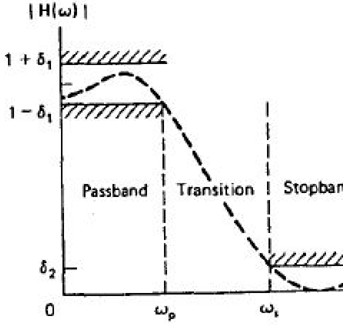


Non Ideal Filters

- As we said the the ideal filters cannot be made in practice
- Moreover sometimes the sharp ending bandpass is not always desirable.
  - For example, if the signals to be separated do not lie in totally disjoint frequency bands.
  - A typical situation to separating them is a gradual transition from passband to stopband.
  - The transition between passband and stop band is named transition band



- A non-ideal low pass filter has three parts: pass band, transition band, stop band
- Deviation from unity of $\pm\delta_1$ is allowed in the passband
- Deviation of δ_2 from zero is allowed in the stopband.
- passband ripple: The amount by which the frequency response differs from unity in the passband
- stopband ripple The amount by which it deviates from zero in the stopband
- ω_p: passband edge; ω_s: stopband edge.
- transition band frequency range from ω_p to ω_s
- Similar definitions are applicable for DT non-ideal filters

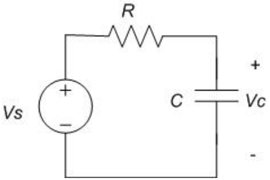


Non-Ideal Filters

- To control the behavior of the filter in time domain, step respond of the filter in investigated.
- The most important and popular indices are:
  - Rise time (t_r) the time for the signal to get to the final value for the first time
  - Overshoot the maximum value minus the step value divided by the step value
  - Settling time the time required for signal to reach and remain within a given error band (5% or 2%) of its final value.

Non-ideal Filters

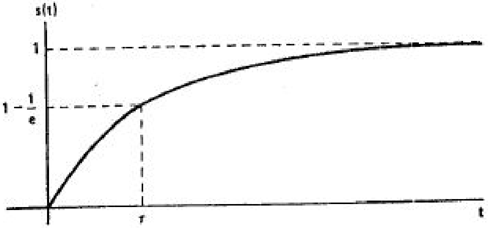
- The ideal filters have great performance in freq. but not acceptable performance in time, they are not implementable.
- Non-ideal filters intend to compromise between freq. performance and time performance.
- A simple example of a non-ideal low pass filter: an RC circuit
  - Input: Voltage source $v_s(t)$; Output: capacitor voltage $V_c(j\omega) = H_L(j\omega)V_s(j\omega)$



A non-ideal low pass filter cont'd

- $RC\frac{dv_c(t)}{dt} + v_c(t) = v_s(t) \Leftrightarrow V_c(j\omega)(RCj\omega + 1) = V_s(j\omega)$
- $\therefore H_L(j\omega) = \frac{V_c(j\omega)}{V_s(j\omega)} = \frac{1}{1+RCj\omega}$

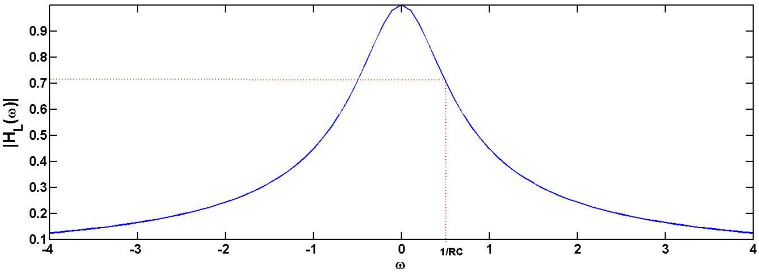
(a)



(b)

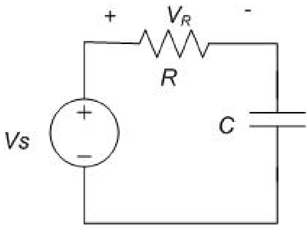
A non-ideal low pass filter cont'd

- $h(t) = \frac{1}{RC} e^{-\frac{t}{RC}} u(t)$
- step response: $s(t) = [1 - e^{-\frac{t}{RC}}]u(t)$
- $\tau = RC$
- To decrease the pass band in freq. $RC\uparrow \rightsquigarrow$ in step response, it takes longer to get to the final value!



A non-ideal high pass filter

- The same RC circuit But
- Voltage source $v_s(t)$; Output: resistor voltage $V_R(j\omega) = H_H(j\omega)V_s(j\omega)$
- $v_R = RC\frac{dv_C}{dt} \rightsquigarrow H_H(j\omega) = \frac{RCj\omega}{1+j\omega RC} = 1 - H_L(j\omega)$

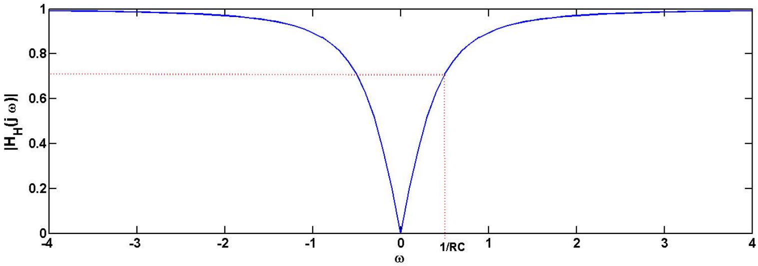


A non-ideal high pass filter cont'd

- Magnitude and phase of freq. response
  - It passes the signal which freq. $|\omega| \geq \frac{1}{RC}$ with min attenuation

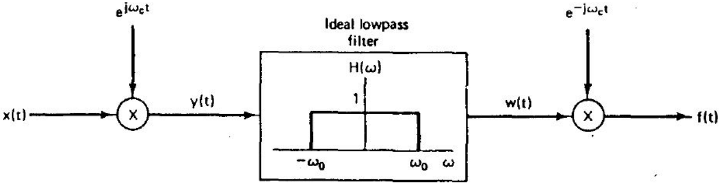
A non-ideal high pass filter cont'd

- Step response of the filter: $s(t) = e^{-\frac{t}{RC}}u(t)$
- By $RC\uparrow$
  - It takes longer time for step response to reach to the final value
  - Pass band of filter is extended (cut of freq. is transferred to the lower freq.)



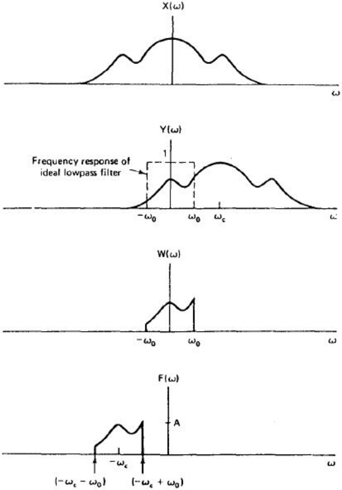
Band Pass Filters

- Band pass filters can also be made by resistors, amplifiers, capacitors, and etc.
- Designing a filter with variable center freq. is more challenging
- One method is designing a filter with fixed freq. and then take advantage of sin amplitude modulation (product property)



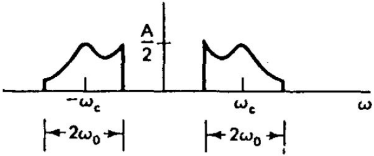
Band Pass Filters

- $Y(j\omega) = \delta(\omega - \omega_c) * X(j\omega) = X(j(\omega - \omega_c))$
- $F(j\omega) = \delta(\omega + \omega_c) * W(j\omega) = W(j(\omega - \omega_c))$

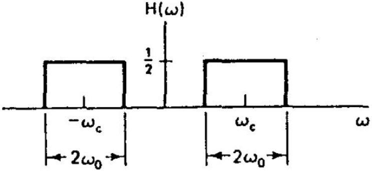


Band Pass Filters

- Now if we keep only real part of f, i.e. use $cos\omega_c t$ instead of $e^{-j\omega_c t}$, we get



- It is equivalent to a bandpass filter with center ω_c



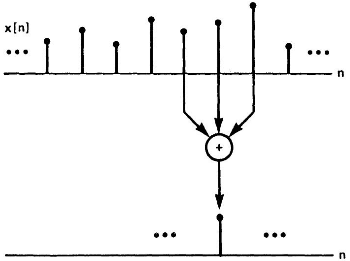
DT Filters

- They are described by difference equations
- The two basic classes:
  - With Recursive equations
  - With Non-recursive equations (Moving Average Filters)

Nonrecursive DT Filters

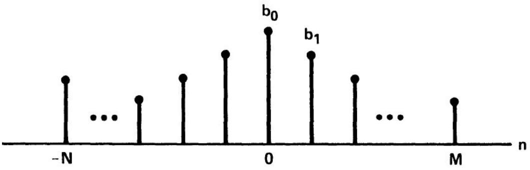
- Consider a Three moving average:

$$y[n] = \frac{1}{3}\{x[n-1] + x[n] + x[n+1]\}$$



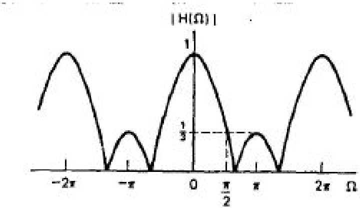
Nonrecursive DT Filters

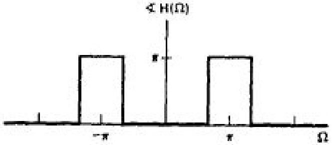
- They have Finite Impulse Response (FIR)
- Their general form is: $y[n] = \sum_{k=-N}^{M} b_k x[n-k]$



Nonrecursive DT Filters

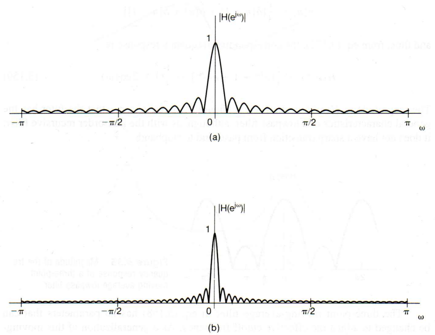
- Example: $M = N = 1$
  $y[n] = \frac{1}{3}(x[n-1] + x[n] + x[n+1])$
  - It is a low pass filter
  - $H(e^{j\omega}) = \frac{1}{3}(e^{j\omega} + 1 + e^{-j\omega}) = \frac{1}{3}(1 + 2cos\omega)$
  - By increasing number of sentences, the obtained filter shape is closer to the deal filter





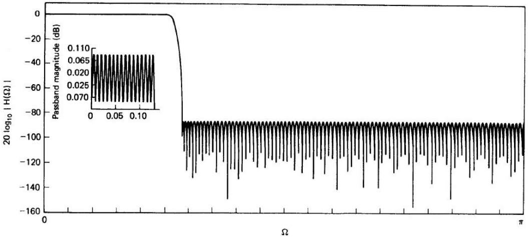
Nonrecursive DT Filters

- Example: Consider FIR filter : $y[n] = \frac{1}{N+M+1}\sum_{k=-N}^{M} x[n-k]$
- Its Freq. Response will be $H(e^{j\omega}) = \frac{1}{N+M+1}\sum_{k=-N}^{M} e^{-j\omega k}$
- Figs show $|H(e^{j\omega})|$ for $N = M = 16$ and 32



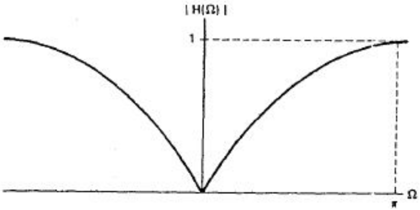
Nonrecursive DT Filters

- This is Freq. Response of a moving average filter with 256 weights



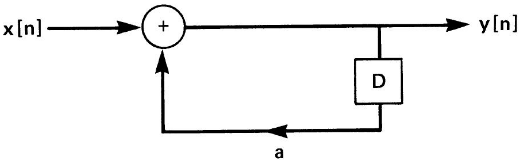
Nonrecursive DT Filters

- Example: $y[n] = \frac{x[n]-x[n-1]}{2}$
  - It is a high pass filter
  - $H(e^{j\omega}) = \frac{1}{2}[1 - e^{-j\omega}] = je^{-j\omega/2} sin\omega/2$
  - To have causal filter N should be negative



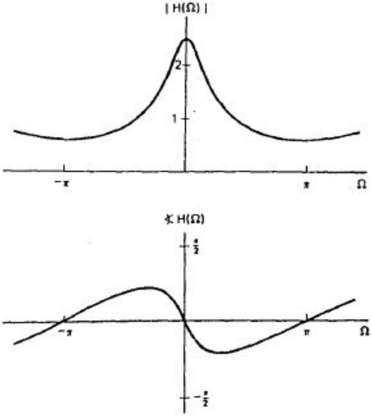
Recursive DT Filters

- Their length of impulse response is infinite (Infinite Impulse Response IIR).
- Their general formula is $\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]$
- First order filter is $y[n] - ay[n-1] = x[n]$



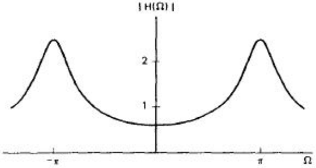
Recursive DT Filters

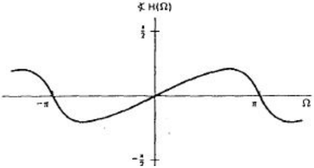
- Example: $y[n] - ay[n-1] = x[n] \rightsquigarrow H(e^{j\omega}) = \frac{1}{1-ae^{-j\omega}}$
- $h[n] = a^n u[n], s[n] = \frac{1-a^{n+1}}{1-a}$
  - By choosing $0 < a < 1$, a low pass filter is obtained



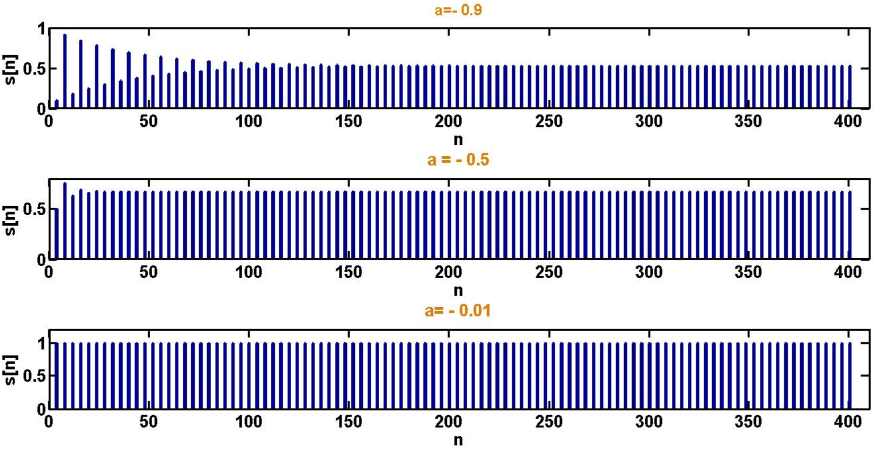
Recursive DT Filter Example Cont'd

- By choosing $-1 < a < 0$, a high pass filter is obtained
- There is a trade off between fast step response in time domain and bandwidth of filter in freq. domain
  - $a\downarrow \rightsquigarrow$ pass band $\downarrow$ and faster response
- Exercise: What will happen if $|a| > 1$ is chosen?





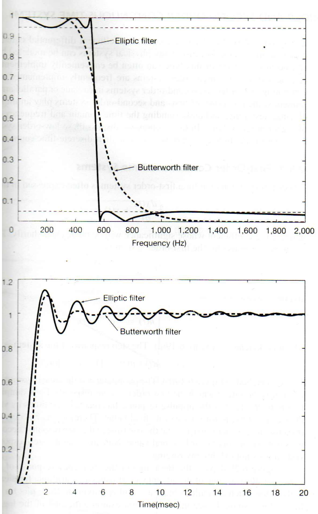
Non-Ideal Filters



Non-Ideal Filters

- By increasing order of filter, sharper filter (from pass band to stop band) with faster response is obtained.
- In designing a lowpass filter a trade of between pass band (freq. domain) and settling time (time domain) can be considered
- Example: The fig. in next page shows a 5th ordered Butterworth filter and a 5th ordered elliptic filter
  - Transient band of elliptic filter is narrower (it is sharper) than Butterworth filter
  - The elliptic filter has more oscillations in step response and its settling time is longer than butterworth filter

Non-Ideal Filters



Non-Ideal Filters

- To obtain sharper filter on can use cascade to identical filters
- $H_1(j\omega) = H_2(j\omega) \rightsquigarrow H(j\omega) = H_1(j\omega)H_2(j\omega) = H_1^2(j\omega)$

Magnitude and Phase of Fourier Transform

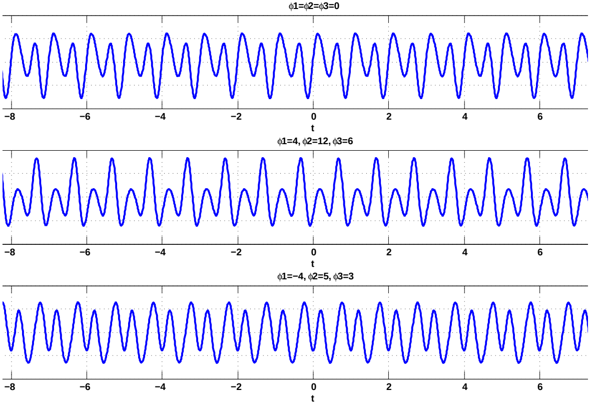
- Fourier Transform is complex in general , therefore it can be expressed in polar representation:
  $X(j\omega) = |X(j\omega)|e^{\measuredangle X(j\omega)}$
  $X(e^{j\omega}) = |X(e^{j\omega})|e^{\measuredangle X(e^{j\omega})}$
- Reconsider Parseval's relation: $\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} \frac{1}{2\pi}|x(j\omega)|^2 d\omega$
  - $|X(j\omega)|^2$ is energy density spectrum of $x(t)$
  - $|X(j\omega)|$ conveys information about relative magnitudes of the complex exponential terms which build $x(t)$
  - $\measuredangle X(j\omega)$ convey information about relative phases of complex exponential terms which build $x(t)$ (phase distortion)

Magnitude-Phase Representation for Freq. response of LTI system

- In general, changes in phase function of $X(j\omega)$ make changes in time domain characteristics of signal $x(t)$
- The auditory system can tolerate phase changes relatively
  - By mild phase distortion in individual sound, the speech is still understandable
  - But severe phase distortion may lead to loose intelligibility
- *Example:* playing a taped record backward
  $\mathcal{F}\{x(-t)\} = X(-j\omega) = |X(j\omega)|e^{-j\measuredangle X(j\omega)}$ (change is only in phase)

Example

- $x(t) = 1 + \frac{1}{2}sin(2\pi t + \phi_1) + sin(4\pi t + \phi_2) + \frac{1}{5}cos(6\pi t + \phi_3)$



Magnitude-Phase Representation for Freq. response of LTI system

- In LTI systems we had: $Y(j\omega) = H(j\omega)X(j\omega) / Y(e^{j\omega}) = H(e^{j\omega})X(e^{j\omega})$
- One can express them in magnitude and phase:
  - $|Y(j\omega)| = |H(j\omega)||X(j\omega)|$
  - $\measuredangle Y(j\omega) = \measuredangle H(j\omega) + \measuredangle X(j\omega)$
  - (similar relation for DT)
- $\therefore$ The effect of an LTI system on input signal is
  - scaling its magnitude by $|H(j\omega)|$ ($|H(j\omega)|$ is gain of the system)
  - adding $\measuredangle H(j\omega)$ to its phase ($\measuredangle H(j\omega)$ is phase shift of the system)
- By designing $H(j\omega)$ properly one can modify the phase and magnitude of input signals (idea of designing controller)

Group Delay

- Consider an LTI system with freq. response: $H(j\omega) = e^{-j\omega t_0}$
- $\rightsquigarrow |H(j\omega)| = 1, and \measuredangle H(j\omega) = -\omega t_0$
- It makes a time shifting or delay: $y(t) = x(t - t_0)$
- A delay in time has negative slop of phase at freq.

Log Magnitude and Bode plots

- To be able to express the magnitude relation of an LTI system by additive terms (similar to phase)logarithmic amplitude can be used: $log|Y(j\omega)| = log|H(j\omega)| + log|X(j\omega)|$
- Logarithmic scale provides this opportunity to display the details in wider dynamic range
- By logarithmic representation cascade of two LTI systems can be expressed as:
  - $log|H(j\omega)| = log|H_1(j\omega)| + log|H_2(j\omega)|$
  - $\measuredangle H(j\omega) = \measuredangle H_1(j\omega) + \measuredangle H_2(j\omega)$
- Since $|H(j\omega)| = |H(-j\omega)|$ and $\measuredangle H(j\omega) = -\measuredangle H(-j\omega)$:
  - For CT log representation is found for $\omega > 0$
  - For DT log representation is found for $0 < \omega < \pi$

FLog Magnitude and Bode plots

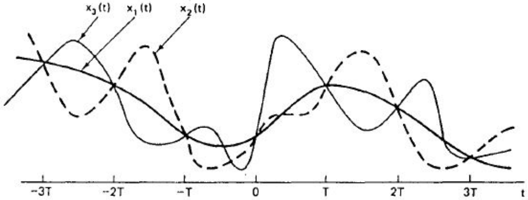
- Unit of logarithm amplitude scale is $20log_{10}$ referred to 1 decibels (1 dB).
  - The name is in honor of Graham Bell
  - It is defined based on the power relation of system $(10log_{10}|H(j\omega)|^2)$
  - Therefore:

$$\begin{aligned}
|H(j\omega)| = 1 \rightarrow &\quad 0dB \\
|H(j\omega)| = \sqrt{2} \rightarrow &\quad \sim 3dB \\
|H(j\omega)| = 2 \rightarrow &\quad \sim 6dB \\
|H(j\omega)| = 10 \rightarrow &\quad 20dB \\
|H(j\omega)| = 100 \rightarrow &\quad 40dB
\end{aligned}$$

- In CT, the freq is also represented by log scale
- Bode plots: Plots of $20log_{10}|H(j\omega)|$ and $\measuredangle H(j\omega)$ versus $log_{10}\omega$
- In DT since the freq. axis is always between $\omega = 0$ and $\omega = \pi$ freq. does not required log scale.
- In some cases like ideal filters which amplitude is none zero only in a limited range of freq. linear scale is more suitable.

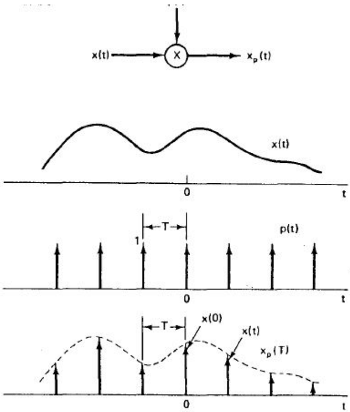
Sampling

- Due to significant development of digital technology, DT processors are more flexible comparing to CT ones.
- We are looking to define a method to transfer CT signals to DT.
- A method is sampling from CT signals
- If we take samples with unified distance from a CT signal, can we always retrieve it uniquely?



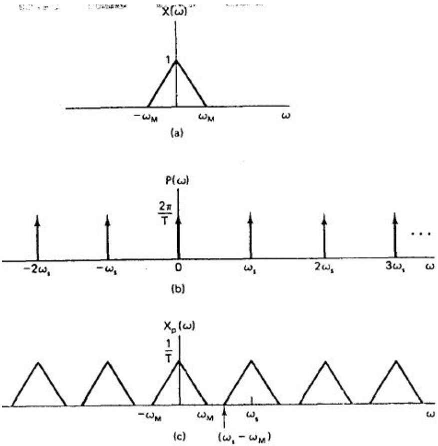
Sampling

- Let us use impulse train to take samples from $x(t)$ in identical distance.
- $p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT) \rightsquigarrow P(j\omega) = \frac{2\pi}{T}\delta(\omega - k\omega_s)$ ($\omega_s = \frac{2\pi}{T}$: sampling freq.)
- $X_p(j\omega) = \frac{1}{2\pi}[X(j\omega) * P(j\omega)]$



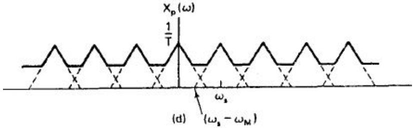
Sampling Effect in Freq.

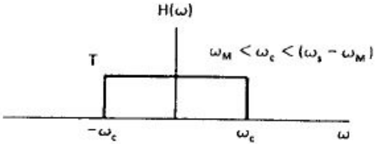
- $X_p(j\omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X(j(\omega - k\omega_s))$
- Assume $\omega_M < \omega_s - \omega_M \rightsquigarrow \omega_s > 2\omega_M$
- $\therefore$ there is no overlap between the shifted replicas of $X(\omega)$



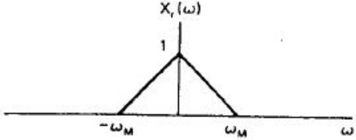
Sampling Effect in Freq.

- If $\omega_s < 2\omega_M$





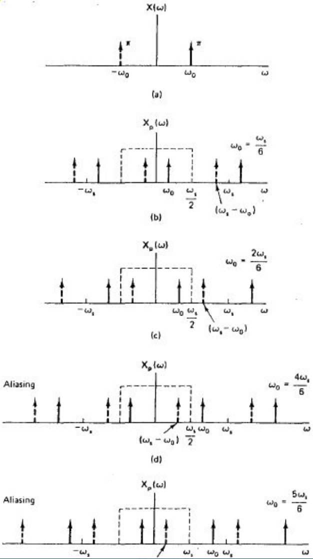
- if $\omega_s > 2\omega_M$, $x(t)$ can be exactly recovered from $x_p(t)$ by employing a lowpass filter with gain T and a cutoff freq. $\omega_M < \omega_c < \omega_s - \omega_m$



Sampling Theorem

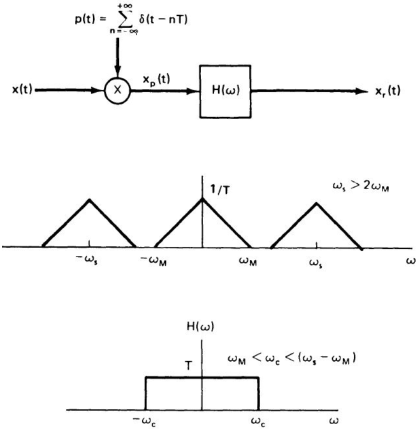
- Let $x(t)$ be a band limited signal with $X(j\omega) = 0$ for $|\omega| > \omega_M$ Then $x(t)$ is uniquely determined by its samples $x(nT), n = 0, \pm 1, \pm 2, \ldots$ if $\omega_s > 2\omega_M$ where $\omega_s = \frac{2\pi}{T}$ Given these samples, we can reconstruct $x(t)$ by generating a periodic impulse train in which successive impulses have amplitudes that are successive sample values. This impulse train is then processed through an ideal lowpass filter with gain T and cutoff frequency $\omega_M < \omega_c < \omega_s - \omega_m$. The resulting output signal will exactly equal $x(t)$.
- ω_s is Nyquist freq.
- ω_M is Nyquist rate

Aliasing


X(ω)
π
π
−ω0
ω0
ω
(a)
Xp(ω)
ω0 = ωs/6
−ωs
ω0
ωs/2
ωs
ω
(ωs − ω0)
(b)
Xp(ω)
ω0 = 2ωs/6
−ωs
ω0
ωs/2
ωs
ω
(ωs − ω0)
(c)
Aliasing
Xp(ω)
ω0 = 4ωs/6
−ωs
(ωs − ω0)
ωs/2
ω0
ωs
ω
(d)
Aliasing
Xp(ω)
ω0 = 5ωs/6
−ωs
ωs
ω0
ωs
ω

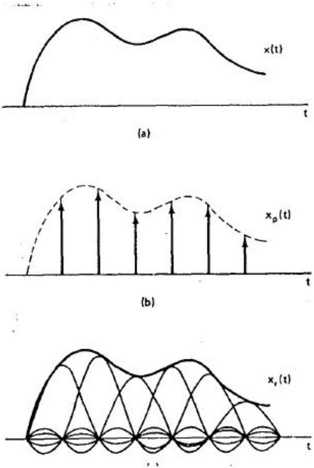
Signal Reconstruction (Interpolation)

- Bandlimited Interpolation: Assuming the signal is bandlimited.
- Interpolation is done by an ideal lowpass filter



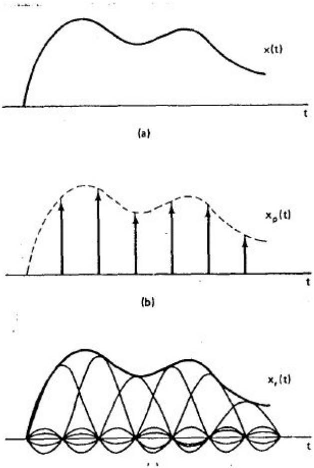
Signal Reconstruction: with Ideal Lowpass Filter

- In time domain:
  $x_p(t) = x(t)\sum_{n=-\infty}^{\infty}\delta(t-nT) = \sum_{n=-\infty}^{\infty}x(nT)\delta(t-nT)$
-
-



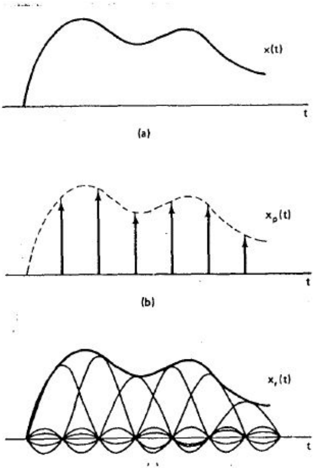
Signal Reconstruction: with Ideal Lowpass Filter

- In time domain:
  $x_p(t) = x(t)\sum_{n=-\infty}^{\infty}\delta(t-nT) = \sum_{n=-\infty}^{\infty} x(nT)\delta(t-nT)$
- $h(t) = \frac{T\omega_0}{\pi} sinc(\frac{\omega_0 t}{\pi})$
-

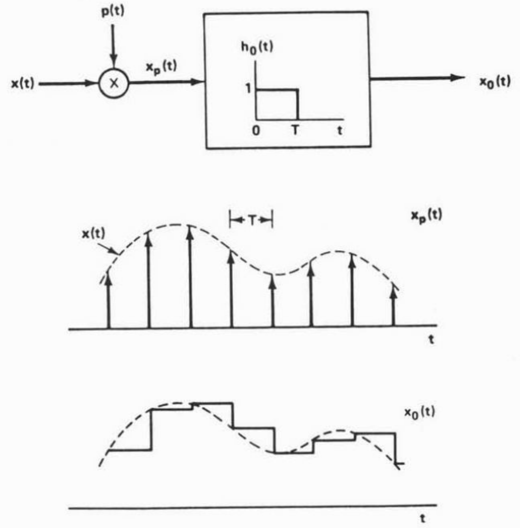


Signal Reconstruction: with Ideal Lowpass Filter

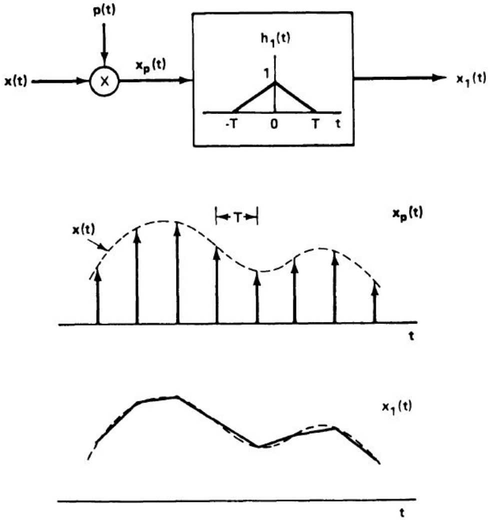
- In time domain:
  $x_p(t) = x(t)\sum_{n=-\infty}^{\infty}\delta(t-nT) = \sum_{n=-\infty}^{\infty}x(nT)\delta(t-nT)$
- $h(t) = \frac{T\omega_0}{\pi}sinc(\frac{\omega_0 t}{\pi})$
- $x_r(t) = x_p(t) * h(t) = \sum_{n=-\infty}^{\infty}x(nT)h(t-nT)$



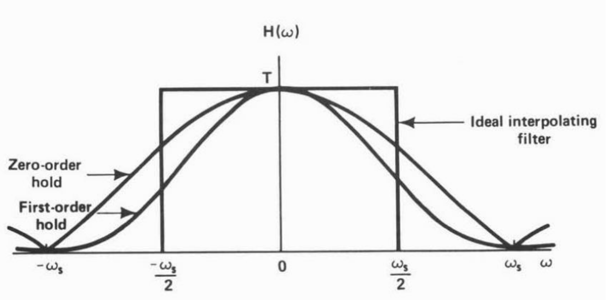
Zero order Hold (ZOH): A Staircase-Like Approximation



First order Hold: A Linear Interpolation



Signal Reconstruction



Original Image

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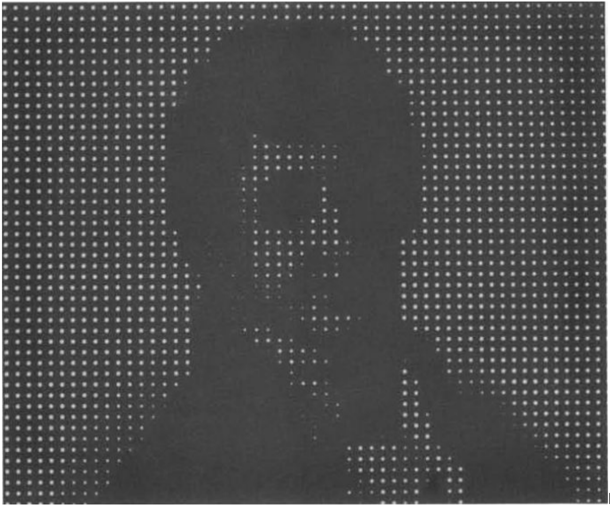
Sampled Image



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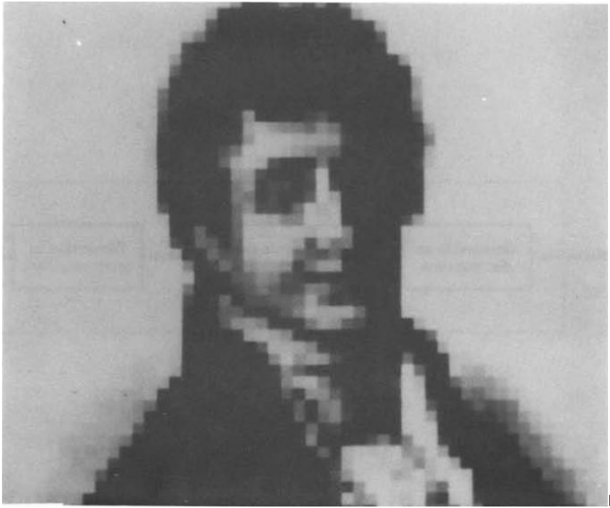
Reconstructing by Zero-Order Hold



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Reconstructing by First-Order Hold

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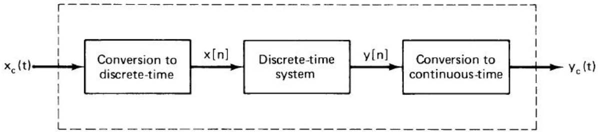
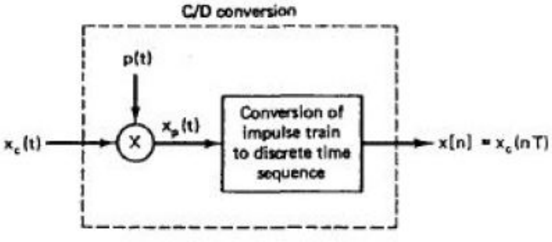
DT Processing of CT Signals



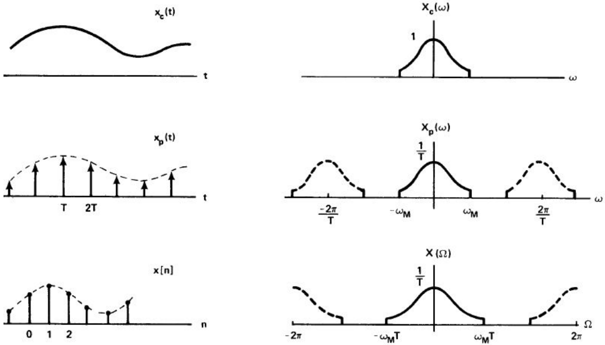
- It is done in three
  1. Continues to Discrete (C/D) Conversion
  2. DT Processing
  3. Discrete to Continues (D/C) Conversion

C/D Converter

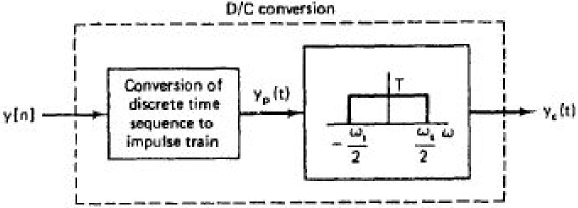


- It is done in two steps:
  1. Sampling:
     $x_p(t) = x_c(t)\sum_{n=-\infty}^{\infty}\delta(t-nT) = \sum_{n=-\infty}^{\infty}x_c(nT)\delta(t-nT)$
  2. Conversion of impulse train to DT sequence:
     - Take CT FT of x_p: $X_p(j\omega) = \sum_{n=-\infty}^{\infty}x_c(nT)e^{-jn\omega T}$
     - Take DT of $x[n]$: $X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty}x[n]e^{-jn\Omega}$
     - $\therefore X(e^{j\Omega}) = X_p(j\omega)|_{\omega T=\Omega}$

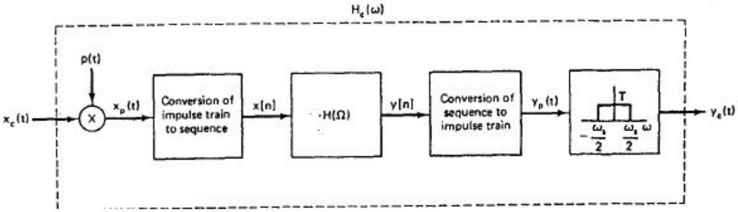
C/D Converter

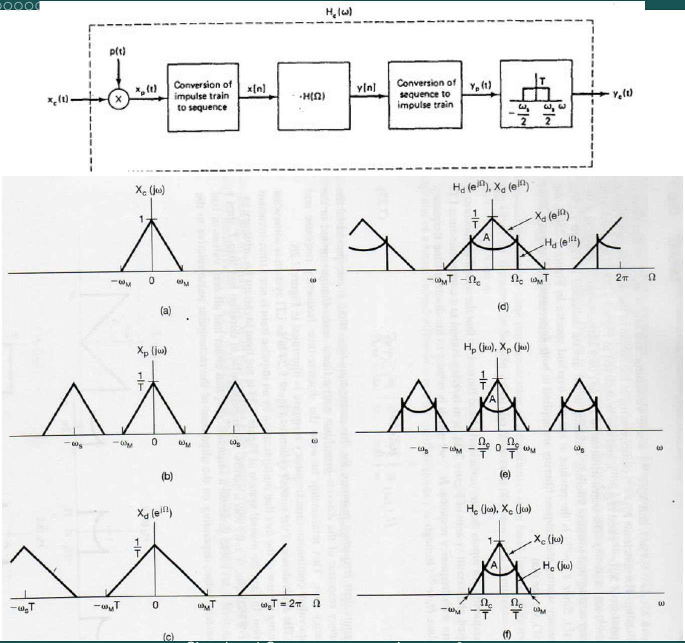


D/C Converter

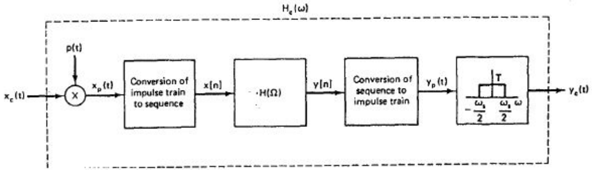


DT Processing of CT Signals




$H_c(\omega)$
$p(t)$
$x_c(t)$
$x_p(t)$
Conversion of impulse train to sequence
$x[n]$
$H(\Omega)$
$y[n]$
Conversion of sequence to impulse train
$y_p(t)$
T
$-\frac{\omega_s}{2}$ $\frac{\omega_s}{2}$ ω
$y_c(t)$
$X_c(j\omega)$
(a)
$X_p(j\omega)$
(b)
$X_d(e^{j\Omega})$
$\omega_s T = 2\pi$
(c)
$H_d(e^{j\Omega}), X_d(e^{j\Omega})$
(d)
$H_p(j\omega), X_p(j\omega)$
(e)
$H_c(j\omega), X_c(j\omega)$
(f)

DT Processing of CT Signals



- $H_c(j\omega) = \begin{cases} H_d(e^{j\omega}) & |\omega| < \omega_s/2 \\ 0 & |\omega| > \omega_s/2 \end{cases}$