Computational Intelligence
Lecture 5:Fuzzy Logic

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Classical Logic

- Logic is the study of methods and principles of reasoning
  - reasoning means obtaining new propositions from existing propositions.
- In classical logic,
  - The propositions are evaluated by true or false.
  - The relationships between propositions are usually expressed by a truth table.
- Logic Formulas: is obtained by combining $-$, $\bigvee$ and $\bigwedge$ in appropriate algebraic expressions
- Tautology: the always true proposition represented by a logic formula, regardless of the truth values of the basic propositions participating in the formula
  - Example: $(p \rightarrow q) \leftrightarrow (\bar{p} \vee q)$
- Contradiction: the always false proposition represented by a logic formula, regardless of the truth values of the basic propositions participating in the formula

Classical Logic

- Inference rules: the forms of tautologies which are used for making deductive inferences
- Some commonly used inference rules are:
  - **Modus Ponens:** $(p \wedge (p \rightarrow q)) \rightarrow q$
    - Premise 1: x is A
    - Premise 2:IF x is A THEN y is B
    - Conclusion: y is B
  - **Modus Tollens:** $(\bar{q} \wedge (p \rightarrow q)) \rightarrow \bar{p}$
    - Premise 1: y is not B
    - Premise 2:IF x is A THEN y is B
    - Conclusion: x is not A
  - **Hypothetical Syllogism:** $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$
    - Premise 1: IF x is A THEN y is B
    - Premise 2: IF y is B THEN z is C
    - Conclusion: IF x is A THEN z is C

- **In fuzzy logic**
  - The propositions are fuzzy propositions that are evaluated by memberships between 0 and 1.
  - The ultimate goal is to provide foundations for approximate reasoning with imprecise propositions
- Consider A, A', B, B' are fuzzy sets
- The fundamental principles are
  - **Generalized Modus Ponens**:
    - Premise 1: x is A'
    - Premise 2: IF x is A THEN y is B
    - Conclusion y is B s.t. the closer A' to A $\rightsquigarrow$ the closer B' to B
    - A' and B' can be

	x is A' (Premise 1)	y is B' (Conclusion)
p1	x is A	y is B
p2	x is very A	y is very B
p3	x is very A	y is B
p4	x is more or less A	y is more or less B
p5	x is more or less A	y is B
p6	x is not A	y is unknown
p7	x is not A	y is not B

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  - If a causal relation between "x is A" and "y is B" is not strong in Premise 2, the satisfaction of $p3$ and $p5$ is allowed.
  - $p7$ is based on "IF x is A THEN y is B, ELSE y is not B."

Fuzzy Logic

- **Generalized Modus Tollens:**
  - Premise 1: y is B'
  - Premise 2: IF x is A THEN y is B
  - Conclusion x is A' s.t. the more different B from B' $\rightsquigarrow$ the more different A from A'
  - A' and B' can be

	y is B' (Premise 1)	x is A' (Conclusion)
t1	y is B	x is A
t2	y is not very B	x is very not A
t3	y is not more or less B	x is not more or less A
t4	y is B	x is unknown
t5	y is not B	x is not A

Generalized Hypothetical Syllogism:

- Premise 1: IF x is A THEN y is B
- Premise 2: IF y is B' THEN z is C
- Conclusion: IF x is A THEN z is C' s.t. the closer B to B' $\rightsquigarrow$ the closer C to C'
- A' and B' can be

	y is B' (Premise 1)	z is C' (Conclusion)
s1	y is B	z is C
s2	y is very B	z is more or less C
s3	y is very B	z is C
s4	y is more or less B	z is very C
s5	y is more or less B	z is C
s6	y is not B	z is unknown
s5	y is not B	z is not C

- For $s2$
  1. change Premise 1 to IF x is very A THEN y is very B
  2. $\therefore$ in Conclusion: IF x is very A THEN z is C
  3. To cancel very, use more or less
  4. $\therefore$ IF x is A THEN z is more or less C

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- Premise 1: IF x is A THEN y is B
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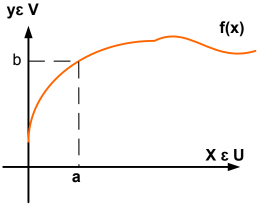
  4. $\therefore$ IF x is A THEN z is more or less C
  - The mentioned intuitive criteria are based on approximate reasoning used in daily life They are not necessarily true for classical cases

- How do we determine the membership functions of the fuzzy propositions in the conclusions?
- **The Compositional Rule of Inference**
  - It is a generalization of the following procedure
    - For a curve $y = f(x)$ from $x \in U$ to $y \in V$
    - $x = a$ and $y = f(x) \rightsquigarrow y = b = f(a)$.
    - Now assume a is an interval and $f(x)$ is an interval-valued function
    - First find a cylindrical set a_E with base a
    - find I: intersection of A_E with the interval-valued curve.
    - The interval b: project I on V



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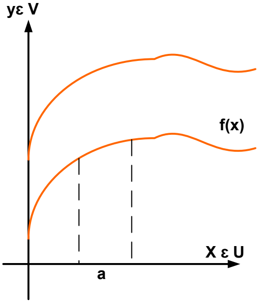

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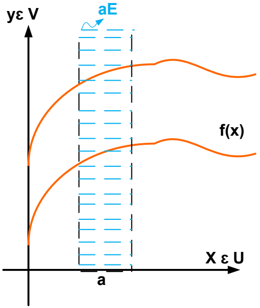

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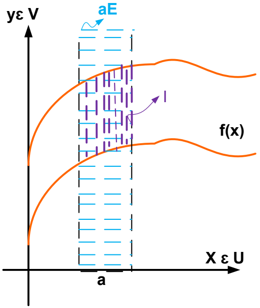

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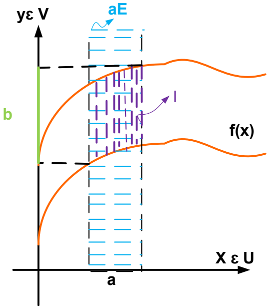

Compositional Rule of Inference.

- Assume the A' is a fuzzy set in U and Q is a fuzzy relation in $U \times V$.
- Then A'_E is cylindrical extension of A': $\mu_{A'_E}(x,y) = \mu_{A'}(x)$
- $I = A'_E \cap Q \rightsquigarrow \mu_I = t\{\mu_{A'_E}(x,y), \mu_Q(x,y)\} = t\{\mu_{A'}(x), \mu_Q(x,y)\}$
- B' proj. of I on V : $\mu_{B'}(y) = \sup_{x \in U} t\{\mu_{A'}(x), \mu_Q(x,y)\}$
- It is compositional rule of inference.
- **Generalized Modus Ponens:**
  - Fuzzy set A':premise x is A'; fuzzy relation $A \rightarrow B \in U \times V$: premise IF x is A THEN y is B; fuzzy set $B' \in V$: conclusion y is B'
    $\mu_{B'}(y) = \sup_{x \in U} t[\mu_{A'}(x), \mu_{A \rightarrow B}(x,y)]$
- **Generalized Modus Tollens:**
  - Fuzzy set B':premise y is B'; fuzzy relation $A \rightarrow B \in U \times V$: premise IF x is A THEN y is B; fuzzy set $A' \in U$: conclusion x is A'
    $\mu_{A'}(x) = \sup_{y \in V} t[\mu_{B'}(y), \mu_{A \rightarrow B}(x,y)]$

- **Generalized Hypothetical Syllogism:**
  - Fuzzy relation $A \rightarrow B \in U \times V$: premise IF x is A THEN y is B; Fuzzy relation $B' \rightarrow C \in V \times W$: premise IF y is B' THEN z is C; Fuzzy relation $A \rightarrow C' \in U \times W$: conclusion IF x is A THEN z is C'; $\mu_{A \rightarrow C'}(x, z) = \sup_{y \in V} t[\mu_{A \rightarrow B}(x, y), \mu_{B' \rightarrow C}(y, z)]$
- Diff. implication principles, definitions of B', A', C' and diff t-norms yields diff. results
- **Generalized Modus Ponens**:
  1. t-norm: min; Mamdani product imp.
     1.1 $A' = A \rightsquigarrow \mu_{B'}(y) = \sup_{x \in U}[\mu_A(x)\mu_B(y)] = \mu_B(y)$
     1.2 $A'=$ very $A \rightsquigarrow \mu_{B'} = \sup_{x \in U}\{\min[\mu_A^2(x), \mu_A(x)\mu_B(y)]\}$
     $\sup_{x \in U}\{\mu_A(x)\} = 1$ and x can take any values in U, for any $y \in V, \exists x \in U$ s.t.
     $\mu_A(x) \geq \mu_B(y) \rightsquigarrow \mu_{B'}(y) = \sup_{x \in U}[\mu_A(x)\mu_B(y)] = \mu_B(y)$
     1.3 A' is more or less A
     $\rightsquigarrow \mu_A^{1/2}(x) \geq \mu_A(x) \geq \mu_A(x)\mu_B(x) \rightsquigarrow \mu_{B'}(y) = \mu_B(y)$
     1.4 $A' = \bar{A}$ for fixed $y \in V, \mu_A(x) \uparrow \rightsquigarrow \mu_A(x)\mu_B(y) \uparrow .1 - \mu_A(x) \downarrow$,
     $\sup_{x \in U}$ min is obtained when
     $1 - \mu_A(x) = \mu_A(x)\mu_B(y) \rightsquigarrow \mu_{B'}(y) = \frac{\mu_B(y)}{1+\mu_B(y)}$

- **Generalized Hypothetical Syllogism:**
  - Fuzzy relation $A \to B \in U \times V$: premise IF x is A THEN y is B; Fuzzy relation $B' \to C \in V \times W$: premise IF y is B' THEN z is C; Fuzzy relation $A \to C' \in U \times W$: conclusion IF x is A THEN z is C'; $\mu_{A\to C'}(x,z) = \sup_{y\in V} t[\mu_{A\to B}(x,y), \mu_{B'\to C}(y,z)]$
- Diff. implication principles, definitions of B', A', C' and diff t-norms yields diff. results
- **Generalized Modus Ponens**:
  1. t-norm: min; Mamdani product imp.
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     - 1.2 $A' =$ very $A \rightsquigarrow \mu_{B'} = \sup_{x\in U}\{\min[\mu_A^2(x), \mu_A(x)\mu_B(y)]\}$ $\sup_{x\in U}\{\mu_A(x)\} = 1$ and x can take any values in U, for any $y \in V, \exists x \in U$ s.t. $\mu_A(x) \geq \mu_B(y) \rightsquigarrow \mu_{B'}(y) = \sup_{x\in U}[\mu_A(x)\mu_B(y)] = \mu_B(y)$
     - 1.3 A' is more or less A $\rightsquigarrow \mu_A^{1/2}(x) \geq \mu_A(x) \geq \mu_A(x)\mu_B(x) \rightsquigarrow \mu_{B'}(y) = \mu_B(y)$
     - 1.4 $A' = \bar{A}$ for fixed $y \in V, \mu_A(x) \uparrow \rightsquigarrow \mu_A(x)\mu_B(y) \uparrow .1 - \mu_A(x) \downarrow$, $\sup_{x\in U}$ min is obtained when $1 - \mu_A(x) = \mu_A(x)\mu_B(y) \rightsquigarrow \mu_{B'}(y) = \frac{\mu_B(y)}{1+\mu_B(y)}$

  p1,p3, and p5 are achieved

Generalized Modus Ponens

2 t-norm: min; Zadeh imp., Assume $\sup_{x \in U}[\mu_A(x)] = 1$

2.1 $A' = A$ $\mu_{B'}(y) = \sup_{x \in U} \min\{\mu_A(x), \max[\min(\mu_A(x), \mu_B(y)), 1 - \mu_A(x)]\}$

- $\sup_{x \in U}[\mu_A(x)] = 1 \rightsquigarrow \sup_{x \in U}$ *min* is achieved at $x_0 \in U$ when $\mu_A(x_0) = \max[\min(\mu_A(x_0), \mu_B(y)), 1 - \mu_A(x_0)]$
- If $\mu_A(x_0) < \mu_B(y) \rightsquigarrow \mu_A(x_0) = \max[\mu_A(x_0), 1 - \mu_A(x_0)]$, it is true when $\mu_A(x_0) \geq 0.5$, since $\sup_{x \in U}[\mu_A(x)] = 1 \rightsquigarrow \mu_B(y) > \mu_A(x_0) = 1$ impossible!
- $\mu_A(x_0) \geq \mu_B(y) \rightsquigarrow \mu_A(x_0) = \max[\mu_B(y), 1 - \mu_A(x_0)]$, If $\mu_B(y) < 1 - \mu_A(x_0) \rightsquigarrow \mu_A(x_0) = 1 - \mu_A(x_0) \rightsquigarrow \mu_A(x_0) = 0.5$; If $\mu_B(y) \geq 1 - \mu_A(x_0) \rightsquigarrow \mu_A(x_0) = \max[0.5, \mu_B(y)]$
- $\therefore \mu_{B'}(y) = \mu_A(x_0) = \max[0.5, \mu_B(y)]$

Generalized Modus Ponens

2 t-norm: min; Zadeh imp., Assume $\sup_{x\in U}[\mu_A(x)] = 1$

2.2 $A' =$ very A

$$\mu_{B'}(y) = \sup_{x\in U} \min\{\mu_A^2(x), \max[\min(\mu_A(x), \mu_B(y)), 1-\mu_A(x)]\}$$

- $\sup_{x\in U}[\mu_A(x)] = 1 \rightsquigarrow \sup_{x\in U}$ *min* is achieved at $x_0 \in U$ when $\mu_A^2(x_0) = \max[\min(\mu_A(x_0), \mu_B(y)), 1-\mu_A(x_0)]$
- If $\mu_A(x_0) < \mu_B(y) \rightsquigarrow \mu_A^2(x_0) = \max[\mu_A(x_0), 1-\mu_A(x_0)]$, it is true when $\mu_A(x_0) = 1$, $\rightsquigarrow \mu_B(y) > 1$ impossible!
- $\mu_A(x_0) \geq \mu_B(y) \rightsquigarrow \mu_A^2(x_0) = \max[\mu_B(y), 1-\mu_A(x_0)]$, If $\mu_B(y) < 1-\mu_A(x_0) \rightsquigarrow \mu_A^2(x_0) = 1-\mu_A(x_0) \rightsquigarrow \mu_A(x_0) = \frac{\sqrt{5}-1}{2}, \mu_{B'}(y) = \mu_A^2(x_0) = \frac{3-\sqrt{5}}{2}$; If $\mu_B(y) \geq 1-\mu_A(x_0) \rightsquigarrow \mu_{B'}(y) = \mu_A^2(x_0) = \mu_B(y) \geq \frac{3-\sqrt{5}}{2}$
- $\therefore \mu_{B'}(y) = \mu_A^2(x_0) = \max[\frac{3-\sqrt{5}}{2}, \mu_B(y)]$

Generalized Modus Ponens

2 t-norm: min; Zadeh imp., Assume $\sup_{x\in U}[\mu_A(x)] = 1$

2.3 $A' =$ more or less A

$$\mu_{B'}(y) = \sup_{x\in U}\min\{\mu_A^{1/2}(x), \max[\min(\mu_A(x), \mu_B(y)), 1-\mu_A(x)]\}$$

- $\sup_{x\in U}[\mu_A(x)] = 1 \rightsquigarrow \sup_{x\in U}$ *min* is achieved at $x_0 \in U$ when $\mu_A^{1/2}(x_0) = \max[\min(\mu_A(x_0), \mu_B(y)), 1-\mu_A(x_0)]$
- similar to the previous case If $\mu_A(x_0) < \mu_B(y)$ is impossible!
- $\mu_A(x_0) \geq \mu_B(y) \rightsquigarrow \mu_A^{1/2}(x_0) = \max[\mu_B(y), 1-\mu_A(x_0)]$,
  If $\mu_B(y) < 1-\mu_A(x_0) \rightsquigarrow \mu_A^{1/2}(x_0) = 1-\mu_A(x_0) \rightsquigarrow \mu_A(x_0) = \frac{3-\sqrt{5}}{2}, \mu_{B'}(y) = \mu_A^{1/2}(x_0) = \frac{\sqrt{5}-1}{2}$;
  If $\mu_B(y) \geq 1-\mu_A(x_0) \rightsquigarrow \mu_{B'}(y) = \mu_A^{1/2}(x_0) = \mu_B(y) \geq \frac{\sqrt{5}-1}{2}$
- $\therefore \mu_{B'}(y) = \mu_A^{1/2}(x_0) = \max[\frac{\sqrt{5}-1}{2}, \mu_B(y)]$

2.4 $A' = \bar{A}$

$$\mu_{B'}(y) = \sup_{x\in U}\min\{1-\mu_A(x), \max[\min(\mu_A(x), \mu_B(y)), 1-\mu_A(x)]\}$$

- $\mu_A(x_0) = 0 \rightsquigarrow 1-\mu_A(x_0) = 1$ and $\max[\min(\mu_A(x), \mu_B(y)), 1-\mu_A(x)] = 1$
- $\therefore \mu_{B'}(y) = 1$

p6 is satisfied

Generalized Modus Tollens

1. t-norm: min; Mamdani product imp.
   1.1 $B' = \bar{B} \rightsquigarrow \mu_{A'}(x) = \sup_{y\in V}[1-\mu_B(y), \mu_A(x)\mu_B(y)]$
      - $\sup_{y\in V}$ min is at $y_0 \in V$ s.t. $1-\mu_B(y_0) = \mu_A(x)\mu_B(y_0) \rightsquigarrow \mu_B(y_0) = \frac{1}{1+\mu_A(x)}$
      - $\therefore \mu_{A'}(x) = 1-\mu_B(y_0) = \frac{\mu_A(x)}{1+\mu_A(x)}$

   1.2 $B' =$ is not very $B \rightsquigarrow \mu_{A'}(x) = \sup_{y\in V}\{\min[1-\mu_B^2(y), \mu_A(x)\mu_B(y)]\}$
      - $\sup_{y\in V}$ min is at $y_0 \in V$ s.t. $1-\mu_B^2(y_0) = \mu_A(x)\mu_B(y_0) \rightsquigarrow \mu_B(y_0) = \frac{\sqrt{\mu_A^2(x)+4}-\mu_A(x)}{2}$
      - $\therefore \mu_{A'}(x) = 1-\mu_B(y_0)\mu_A(x) = \frac{\mu_A(x)\sqrt{\mu_A^2(x)+4}-\mu_A^2(x)}{2}$

   1.3 B' is more or less B
   $\rightsquigarrow \mu_{A'}(x) = \sup_{y\in V}\{\min[1-\mu_B^{1/2}(y), \mu_A(x)\mu_B(y)]\}$
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      - $\therefore \mu_{A'}(x) = \mu_A(x)\mu_B(y_0) = \frac{1+2\mu_A(x)-\sqrt{\mu_A^2(x)+1}}{2\mu_A(x)}$

Generalized Modus Tollens

1. t-norm: min; Mamdani product imp.
   1.4 $B' = B \rightsquigarrow \mu_{A'}(x) = \sup_{y \in V}\{\min[\mu_B(y), \mu_A(x)\mu_B(y)]\} = \sup_{y \in V} \mu_B(y)\mu_A(x) = \mu_A(x)$
      - $\therefore \mu_{A'}(x) = \mu_A(x)$
      - $t1$ is satisfied : y is $B \rightsquigarrow x$ is A

Generalized Hypothetical Syllogism

1. t-norm: min; Mamdani product imp.
   1.1 $B' = B \rightsquigarrow \mu_{A\to C'}(x,z) = \sup_{y\in V}\{\min[\mu_A(x)\mu_B(y), \mu_B(y)\mu_C(z)]\} = (\sup_{y\in V}\mu_B(y))\min[\mu_A(x), \mu_C(z)]$
      - $\sup_{y\in V}[\mu_B(y)] = 1 \rightsquigarrow \mu_{A\to C'}(x,z) = \min[\mu_A(x), \mu_C(z)]$
   1.2 $B' = \text{very } B \rightsquigarrow \mu_{A\to C'}(x,z) = \sup_{y\in V}\{\min[\mu_A(x)\mu_B(y), \mu_B^2(y)\mu_C(z)]\}$
      - If $\mu_A(x) > \mu_C(z) \rightsquigarrow \mu_A(x)\mu_B(y) > \mu_B^2(y)\mu_C(z)$
      - $\therefore \mu_{A\to C'}(x,z) = \sup_{y\in V}[\mu_B^2(y)\mu_C(z)] = \mu_C(z)$
      - If $\mu_A(x) \le \mu_C(z) \rightsquigarrow \sup_{y\in V}$ min is at $y_0 \in V, \mu_A(x)\mu_B(y_0) = \mu_B^2(y_0)\mu_C(z)$
      - $\therefore \mu_B(y_0) = \frac{\mu_A(x)}{\mu_C(z)} \rightsquigarrow \mu_{A\to C'}(x,z) = \mu_A(x)\mu_B(y_0) = \frac{\mu_A^2(x)}{\mu_C(z)}$
      - $\therefore \mu_{A\to C'}(x,z) = \begin{cases} \mu_C(z) & \text{if } \mu_C(z) < \mu_A(x) \\ \frac{\mu_A^2(x)}{\mu_C(z)} & \text{if } \mu_C(z) \ge \mu_A(x) \end{cases}$

Generalized Hypothetical Syllogism

1. t-norm: min; Mamdani product imp.

   1.3 $B' =$ more or less B

   $\rightsquigarrow \mu_{A\to C'}(x,z) = \sup_{y\in V}\{\min[\mu_A(x)\mu_B(y), \mu_B^{1/2}(y)\mu_C(z)]\}$

   - Using similar method to $B' =$very B
   - $\mu_{A\to C'}(x,z) = \begin{cases} \mu_A(x) & \text{if } \mu_A(x) < \mu_C(z) \\ \frac{\mu_C^2(z)}{\mu_A(x)} & \text{if } \mu_A(x) \geq \mu_C(z) \end{cases}$

   1.4 $B' = \bar{B} \rightsquigarrow \mu_{A\to C'}(x,z) = \sup_{y\in V}\{\min[\mu_A(x)\mu_B(y), (1-\mu_B(y))\mu_C(z)]\}$

   - $\sup_{y\in V}$ min is achieved at $\mu_B(y_0) = \frac{\mu_C(z)}{\mu_A(x)+\mu_C(z)}$
   - $\therefore \mu_{A\to C'}(x,z) = \frac{\mu_A(x)\mu_C(z)}{\mu_A(x)+\mu_C(z)}$