Computational Intelligence
Part II
Lecture 3: Identification and Control Design Using Fuzzy Systems

Farzaneh Abdollahi

Department of Electrical Engineering

Amirkabir University of Technology

Identification

- Fuzzy systems using Gradient Descent Method
- Example

Control

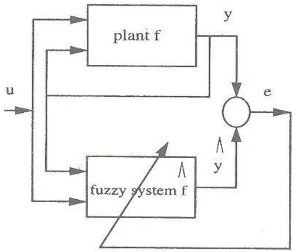
- Indirect Adaptive Fuzzy Control
- Direct Adaptive Fuzzy Control

Identification

- Consider the system dynamics:

$$y(k+1) = f(y(k), ..., y(k-n+1), u(k), \ldots, u(k-m+1))$$

  - u: input; y:output; $f(.)$: an **unknown** function.
  - Open loop system is stable.
- Identification model
  $$\hat{y}(k+1) = \hat{f}(y(k), ..., y(k-n+1), u(k), \ldots, u(k-m+1))$$
  - $\hat{f}$: estimated f; $\hat{y}$: identifier output
- Objective: By using desired pairs of I/O (x^{k+1}, y^{k+1}),identifying f s.t. $e = y - \hat{y}$ is arbitrarily small.
- $x^{k+1} = (y(k), \ldots, y(k-n+1), u(k), \ldots, u(k-m+1))$ obtained by TDL.



Fuzzy systems using Gradient Descent Method

- f is designed based on a fuzzy system
- Its parameters are adjusted by gradient descent method.
- The structure of the identifier can be either parallel or series parallel.
- For example: a fuzzy system including:
  - Inference engine: production
  - Fuzzifier: singleton
  - Difuzzifier: center average
  - Membership function: Gaussian

$$f(x) = \frac{\sum_{l=1}^{M} \bar{y}^l [\prod_{i=1}^{n} \exp(-(\frac{x_i - \bar{x}_i^l}{\sigma_i^l})^2)]}{\sum_{l=1}^{M} [\prod_{i=1}^{n} \exp(-(\frac{x_i - \bar{x}_i^l}{\sigma_i^l})^2)]} \qquad (1)$$

- Unknown parameters: $\bar{x}_i^l, \bar{y}_i^l, \sigma_i^l$

Training

1. Choosing a fuzzy system and initial values:
   - Assume the system fuzzy (1)
   - Choose a proper value for M
     - The greater $M \rightsquigarrow$ More accuracy with complicated structure
   - Choose initial values $\bar{x}_i^l(0), \bar{y}_i^l(0), \sigma_i^l(0)$ randomly, based on linguistic rules or a priori knowledge of the system
2. Apply input and calculate output of The fuzzy system
   - Apply the desired I/O pair $(x(k), y(k)),\ k = 1, 2, \ldots$
   - Calculate f in (1) in following three steps (layers)

     2.1 $z^l = \prod_{i=1}^{n} \exp(-(\frac{x_i(k) - \bar{x}_i^l(k)}{\sigma_i^l(k)})^2)$

     2.2 $b = \sum_{l=1}^{M} z^l$

     2.3 $a = \sum_{l=1}^{M} \bar{y}^l(k) z^l$

     2.4 $\hat{y}(k) = \hat{f} = \frac{a}{b}$

Introduction

3. Updating Paraments
   - Using Gradient decent method find $\sigma_i^l(k+1)$, $\bar{x}_i^l(k+1), \bar{y}_i^l(k+1)$

$$
\begin{aligned}
\bar{y}^l(k+1) &= \bar{y}^l(k) - \eta \frac{\hat{f}-y}{b} z^l, l = 1, ..., M \\
\bar{x}_i^l(k+1) &= \bar{x}_i^l(k) - \eta(\hat{f}-y)\frac{\bar{y}^l-\hat{f}}{b} z^l \frac{2(x_i(k)-\bar{x}_i^l(k))}{\sigma_i^{l2}}, i = 1, \ldots, n \\
\sigma_i^l(k+1) &= \sigma_i^l(k) - \eta \frac{\hat{f}-y}{b} z^l(\bar{y}^l(k)-\hat{f})\frac{2(x_i(k)-\bar{x}_i^l(k))^2}{\sigma_i^{l3}(k)}
\end{aligned}
$$

   - a, b, z_l are found in the second step, $\eta > 0$ is learning rate

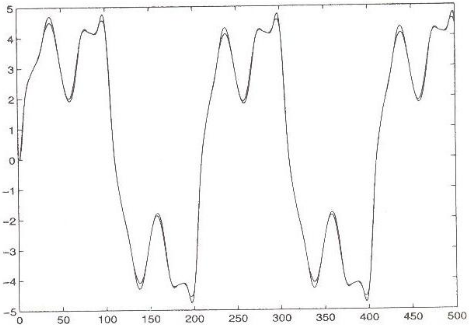
4. $k = k+1$, go back to step 2, and repeat this loop until $|y(k) - \hat{y}(k)|$ is arbitrarily small.

Example

- Identify $y(k+1) = 0.3y(k) + 0.6y(k-1) + g[u(k)]$
- $g(u) = 0.6\sin(\pi u) + 0.4\sin(3\pi u) + 0.1\sin(5\pi u)$ is unknown
- Identification model $\hat{y}(k+1) = 0.3y(k) + 0.6y(k-1) + \hat{g}[u(k)]$
- Choose $M = 10$, $\eta = 0.5$
- $u(k) = \sin(2\pi k/200)$

Example Cont'd

- The outputs of the plant and the model after the identification procedure



Adaptive Fuzzy Control

- The objective of adaptive control: Providing desired performance in presence of uncertainties.
- The main advantage of adaptive fuzzy control comparing to classical adaptive control:
  - To obtain control adaptive law, the knowledge of experts on system dynamics and/or control strategies can be considered.
- Expert knowledge can be categorized to
  - System knowledge: The If-then rules which describe the unknown system behavior.
    - For example: For a car:"IF you push the gas pedal more, Then the car speed is increased.
  - Control knowledge: the rule of fuzzy control which indicates at each situation, which control action is required.
    - For example: For a car:"IF the speed is low, Then push the gas pedal more.

- Based on the applied type of expert knowledge, the adaptive fuzzy control can be
  - Indirect adaptive fuzzy control: The fuzzy control includes some fuzzy systems made based on system knowledge
  - Direct adaptive fuzzy control: The control fuzzy includes a fuzzy system which is made based on control knowledge
  - Combination of indirect/direct adaptive fuzzy control: A weighted combination of direct and indirect adaptive control
- **Indirect Adaptive Fuzzy Control**
  - Consider nth order nonlinear system

$$\begin{aligned} x^{(n)} &= f(x, \dot{x}, \ldots, x^{(n-1)}) + g(x, \dot{x}, \ldots, x^{(n-1)})u \\ y &= x \end{aligned}$$

    where $X = (x, \dot{x}, \ldots, x^{(n-1)})$: state vector; $u \in R$: input; $y \in R$: Output; f, g : unknown functions
  - Assume the system is controllable
  - Objective: find $u = u(X|\theta)$ based on fuzzy rules and an adaptation law for adjusting θ s.t. y tracks y_m

The Fuzzy Control Design for Indirect Adaptive Control

- Assume a set of IF-then laws based on system knowledge is available to describe the I/O behavior of g and f

$$\text{If } x_1 \text{ is } F_1^r, \dots, x_n \text{ is } F_n^r, \text{then } f(x) \text{ is } C^r \tag{2}$$
$$\text{If } x_1 \text{ is } G_1^s, \dots, x_n \text{ is } G_n^s, \text{then } g(x) \text{ is } D^r$$

$r = 1, 2, \dots, L_f$ $s = 1, 2, \dots, L_g$

- If the f and g functions are known, u is selected s.t. cancel the nonlinearities and control based on linear control techniques such as pole-placement:

$$u^* = \frac{1}{g(x)}[-f(x) + y_m^{(n)} + K^T e] \tag{3}$$

where $e = y_m - y$ is dynamics error, $K = (k_1, \dots, k_n)^T$, s.t. the roots of $s^n + k_1 s^{n-1} + \dots + k_n$ are LHP

- Since f and g are unknown, the estimation of them are considered in (4):

$$u^* = \frac{1}{\hat{g}(X|\theta_g)}[-\hat{f}(X|\theta_f) + y_m^{(n)} + K^T e] \tag{4}$$

- $\hat{g}(X|\theta_g)$ and $\hat{f}(X|\theta_f)$ are obtained in the following two steps
  1. for $x_i,\ i=1,\ldots,n$, define p_i fuzzy set of $A_i^{l_i}, l_i=1,\ldots,p_i$, s.t. they include $F_i^r,\ r=1,\ldots,L_f$ in(2); also define q_i fuzzy set of $B_i^{l_i}, l_i=1,\ldots,q_i$, s.t. they include $G_i^s,\ s=1,\ldots,L_g$ in(2)
  2. Using the fuzzy rule $\prod_{i=1}^{n} p_i$ provide a fuzzy system for $\hat{f}(X|\theta_f)$:

$$\text{If}\ \ x_1\ \ \text{is}\ \ B_1^{l_1},\ldots,x_n\ \ \text{is}\ \ A_n^{l_n},\text{then}\ \ \hat{f}(x)\ \text{is}\ E^{l_1,\ldots,l_n}, \tag{5}$$

  for $l_i=1,\ldots,p_i, i=1,\ldots,n$
    - If the If part of (2) is the same as If part of (5), then $E^{l_1,\ldots,l_n}$ is C^r.
    - Otherwise, it is considered ad a new fuzzy set

- Using the fuzzy rule $\prod_{i=1}^{n} q_i$ provide fuzzy system for $\hat{g}(X|\theta_g)$:

$$\text{If}\ \ x_1\ \ \text{is}\ \ A_1^{l_1},\ldots,x_n\ \ \text{is}\ \ B_n^{l_n},\text{then}\ \ \hat{g}(x)\ \text{is}\ H^{l_1,\ldots,l_n} \tag{6}$$

  for $l_i=1,\ldots,q_i, i=1,\ldots,n$
- If the If part of (2) is the same as If part of (6), then $H^{l_1,\ldots,l_n}$ is D^r.
- Otherwise, it is considered ad a new fuzzy set

- Consider:
  Inference engine: production; Fuzzifier: singleton; Difizzifier: center average

$$f(X|\theta_f) = \frac{\sum_{l_1=1}^{p_1}\cdots\sum_{l_n=1}^{p_n}\bar{y}_f^{l_1\ldots l_n}[\prod_{i=1}^{n}\mu_{A_i}{}^{l_i}(x_i)]}{\sum_{l_1=1}^{p_1}\cdots\sum_{l_n=1}^{p_n}[\prod_{i=1}^{n}\mu_{A_i}{}^{l_i}(x_i)]} \quad (7)$$

$$g(X|\theta_g) = \frac{\sum_{l_1=1}^{q_1}\cdots\sum_{l_n=1}^{q_n}\bar{y}_g^{l_1\ldots l_n}[\prod_{i=1}^{n}\mu_{B_i}{}^{l_i}(x_i)]}{\sum_{l_1=1}^{q_1}\cdots\sum_{l_n=1}^{q_n}[\prod_{i=1}^{n}\mu_{B_i}{}^{l_i}(x_i)]} \quad (8)$$

- Consider $\bar{y}_f^{l_1\ldots l_n}$ and $\bar{y}_g^{l_1\ldots l_n}$ are free parameters which are summed in $\theta_f \in R^{\prod_{i=1}^{n} p_i}$ and $\theta_g \in R^{\prod_{i=1}^{n} q_i}$, respectively:

$$f(X|\theta_f) = \theta_f^T\varepsilon(X)$$

$$g(X|\theta_g) = \theta_g^T\eta(X)$$

$$\varepsilon(X) = \frac{\prod_{i=1}^{n}\mu_{A_i}{}^{l_i}(x_i)}{\sum_{l_1=1}^{p_1}\cdots\sum_{l_n=1}^{p_n}[\prod_{i=1}^{n}\mu_{A_i}{}^{l_i}(x_i)]} \quad (9)$$

$$\eta(X) = \frac{\prod_{i=1}^{n}\mu_{B_i}{}^{l_i}(x_i)}{\sum_{l_1=1}^{q_1}\cdots\sum_{l_n=1}^{q_n}[\prod_{i=1}^{n}\mu_{B_i}{}^{l_i}(x_i)]} \quad (10)$$

- Adapting Rule:

$$\begin{aligned} \dot{\theta}_f &= -\gamma_1 e^T P b \varepsilon(X) \\ \dot{\theta}_g &= -\gamma_2 e^T P b \eta(X) \end{aligned} \tag{11}$$

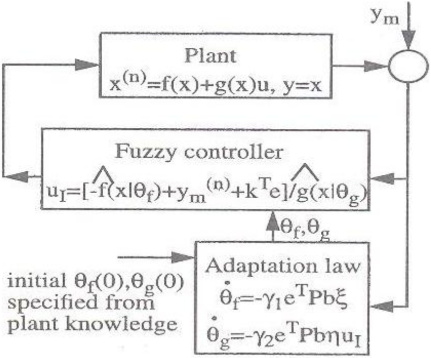
- where $-\gamma_1$, $-\gamma_2$ are pos. numbers and P is Pos. def. matrix obtained from Lyapunov equation

$$\Lambda^T P + P\Lambda = -Q, Q > 0, \quad \Lambda = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & . \\ . & . & . & \dots & . \\ . & . & . & \dots & . \\ 0 & 0 & 0 & \dots & 1 \\ -k_n & -k_{n-1} & \dots & \dots & -k_1 \end{pmatrix}$$

- It should be mentioned that the system knowledge (2) is considered on selecting $\theta_f(0), \theta_g(0)$

Indirect Adaptive Fuzzy Control

- Indirect Adaptive Fuzzy Control



Direct Adaptive Fuzzy Control

- Consider nth order nonlinear system

$$x^{(n)} = f(x, \dot{x}, \ldots, x^{(n-1)}) + bu$$
$$y = x$$

where $X = (x, \dot{x}, \ldots, x^{(n-1)})$: state vector; $u \in R$: input; $y \in R$: Output; f : unknown functions, $b > 0$ is cons. and unknown

- Assume the system is controllable
- Objective: find $u = u(X|\theta)$ based on fuzzy rules and an adaptation law for adjusting θ s.t. y tracks y_m
- Main difference of direct and indirect adaptive fuzzy control is type of available expert knowledge
- In direct adaptive fuzzy control, assume a set of IF-then laws based on *control knowledge*

$$\text{If } x_1 \text{ is } P_1^r, \ldots, x_n \text{ is } P_n^r, \text{then } u \text{ is } Q^r, \quad r = 1, 2, \ldots, L_u \tag{12}$$

The Fuzzy Control Design

- $u_D(X|\theta_f)$ is obtained in the following two steps
  1. for $x_i,\ i = 1, \ldots, n$, define m_i fuzzy set of $A_i^{l_i}, l_i = 1, \ldots, m_i$, s.t. they include $p_i^r,\ r = 1, \ldots, L_u$ in(12)
  2. Using the fuzzy rule $\prod_{i=1}^{n} m_i$ provide fuzzy system for $u(X|\theta_u)$:

$$\text{If } \ x_1 \ \text{ is } \ A_1^{l_1}, \ldots, x_n \ \text{ is } \ A_n^{l_n}, \text{then } \ u \text{ is } S^{l_1,\ldots,l_n}, \tag{13}$$

for $l_i = 1, \ldots, m_i, i = 1, \ldots, n$

  - If the If part of (13) is the same as If part of (5), then $E^{l_1,\cdots,l_n}$ is C^r.
  - Otherwise, it is considered ad a new fuzzy set

- Consider: Inference engine: Production; Fuzzifier: singleton; Difizzifier: center mean

$$u(X|\theta_f) \quad = \quad \frac{\sum_{l_1=1}^{m_1} \cdots \sum_{l_n=1}^{m_n} \bar{y}_u^{l_1 \ldots l_n} [\prod_{i=1}^{n} \mu_{A_i}{}^{l_i}(x_i)]}{\sum_{l_1=1}^{m_1} \cdots \sum_{l_n=1}^{m_n} [\prod_{i=1}^{n} \mu_{A_i}{}^{l_i}(x_i)]} \tag{14}$$

- Consider $\bar{y}_u^{l_1\cdots l_n}$ are adjustable parameters, summed in $\theta_u \in R^{\prod_{i=1}^{n} p_i}$:

$$
\begin{aligned}
u(X|\theta_u) &= \theta_u^T \varepsilon(X) \\
\varepsilon(X) &= \frac{\prod_{i=1}^{n} \mu_{A_i}{}^{l_i}(x_i)}{\sum_{l_1=1}^{m_1} \cdots \sum_{l_n=1}^{m_n} [\prod_{i=1}^{n} \mu_{A_i}{}^{l_i}(x_i)]}
\end{aligned} \tag{15}
$$

- Adapting Rule:

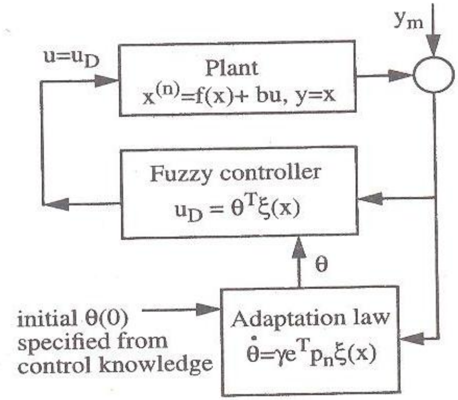
$$\dot{\theta}_u = \gamma_3 e^T P_n \varepsilon(X)$$

- where $-\gamma_3$, is pos. numbers and p_n is the last column of P is Pos. def. matrix obtained from Lyapunov equation

$$\Lambda^T P + P\Lambda = -Q, Q > 0, \quad \Lambda = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & . \\ . & . & . & \dots & . \\ . & . & . & \dots & . \\ 0 & 0 & 0 & \dots & 1 \\ -k_n & -k_{n-1} & \dots & \dots & -k_1 \end{pmatrix}$$

Direct Adaptive Fuzzy Control

- Direct Adaptive Fuzzy Control



Example

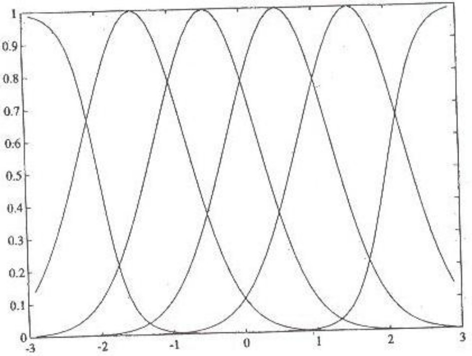
- Consider a system dynamics: $\dot{x} = \frac{1-e^{-x(t)}}{1+e^{-x(t)}} + u(t)$
- Objective is finding a controller s.t. $x \to 0$.
- choose $\gamma_3 = 1$, and six fuzzy sets $N_1, N_2, N_3, p_1, p_2, p_3$ in $[-3, 3]$
- Membership fucns

$$\mu_{N_1}(x) = \exp(-(x+0.5)^2), \mu_{N_2}(x) = \exp(-(x+1.5)^2),$$
$$\mu_{p_1}(x) = \exp(-(x-2)^2), \mu_{p_2}(x) = \exp(-(x+1.5)^2)$$
$$\mu_{N_3}(x) = \exp(-(x+2)^2), , \mu_{p_3}(x) = \exp(-(x-0.5)^2)$$

- To cases are considered
  - There is no control fuzzy rule, $\theta_i(0)$ is obtained randomly in $[-2, 2]$
    - If x is N_2, then $u(x)$ is PB (if $x < 0$, choose $u >> 0$ to make $\dot{x} > 0$)
    - If x is P_2, then $u(x)$ is NB (if $x > 0$, choose $u << 0$ to make $\dot{x} < 0$)
    - where $\mu_{NB}(u) = \exp(-(u+2)^2), \mu_{PB}(u) = \exp(-(u-2)^2)$

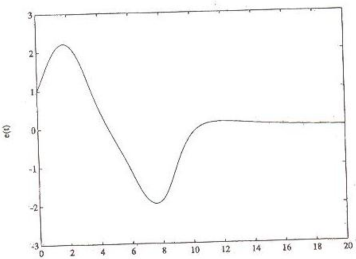
Example Cont'd

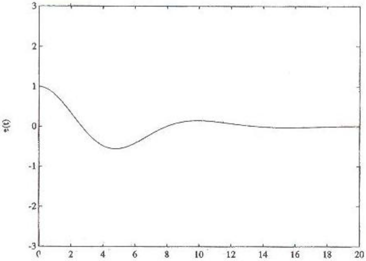
- Membership Function



Example Cont'd

- x in closed–loop system using direct control fuzzy with a) unknown fuzzy control rules; b) known fuzzy control rules
- in (b) the state converges faster





References

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