Computational Intelligence
Lecture 17: Fuzzy Control I

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Comparing Fuzzy Control with Conventional Control

- Similarities:
  - They must address the same issues that are common to any control problem, e.g., stability and performance.
  - The mathematical tools used to analyze the designed control systems are similar, because they are studying the same issues (stability, convergence, etc.) for the same kind of systems.
- Difference
  - In conventional control mathematical model of the process and controllers are available. In fuzzy control, the controllers are designed using rules based on heuristics and human expertise
    - Advanced fuzzy controllers may use both heuristics and mathematical models

Fuzzy Control

- Fuzzy control is classified into
  - Nonadaptive Fuzzy Control
    - the structure and parameters of the fuzzy controller are fixed
  - Adaptive Fuzzy Control
    - The structure or/and parameters of the fuzzy controller change during realtime operation.
- Nonadaptive fuzzy control is simpler than adaptive fuzzy control
- Nonadaptive fuzzy control requires more knowledge of the process model or heuristic rules.
- Adaptive requires less information and may perform better at the cost of more complexity.

Assumption is Fuzzy Control Design

- The plant is observable and controllable: state, input, and output variables are usually available for observation and measurement or computation.
- There exists a body of knowledge comprised of a set of linguistic rules, engineering common sense, intuition, or a set of inputoutput measurements data from which rules can be extracted
- A solution exists.
- The control engineer is looking for a good enough solution, not necessarily the optimum one.
- The controller will be designed within an acceptable range of precision.

The Trial-and-Error Approach

- By using experience-based knowledge (e.g., an operating manual) and by asking the domain 'experts to answer a carefully organized questionnaire, IF-THEN rules are provided and fuzzy controllers are constructed
- Then the fuzzy controllers are tested in the real system and if the performance is not satisfactory, the rules are fine-tuned or redesigned in a number of trial-and-error cycles until the desired performance is achieved.

The Trial-and-Error Approach

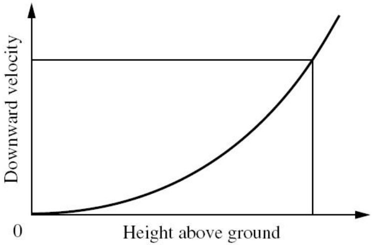
1. Analyze the real system to choose state and control variables and outputs.
   - The state variables:
     - characterize the key features of the system
   - The control variables (inputs of the plant):
     - influence the states of the system.
     - are the outputs of the fuzzy controller.
2. Partition the universe of discourse or the interval spanned by each variable into a number of fuzzy subsets, assigning each a linguistic label
   - Assign or determine a membership function for each fuzzy subset.
   - You may require to choose appropriate scaling factors for the input and output variables in order to normalize the variables to the $[0, 1]$ or the $[-1, 1]$ interval.

The Trial-and-Error Approach

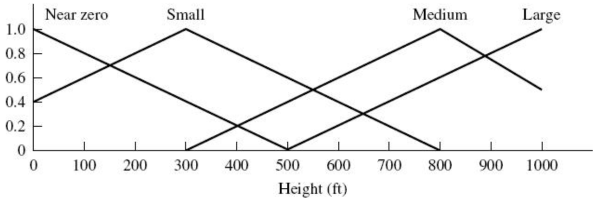
3. Derive IF-THEN rules that relate the state variables with the control variables
    - The rules are defined using
        - An introspective verbalization of human expertise like operating manual
        - the information obtained from a filled carefully organized questionnaire
4. Design the fuzzy system and test the closed-loop system with this fuzzy system as the controller
    - If the performance is not satisfactory, fine-tune or redesign the fuzzy controller by trial and error
    - repeat the procedure until achieving the desired performance

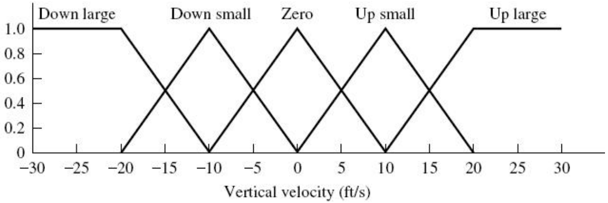
Example:AIRCRAFT LANDING CONTROL

- The desired profile is shown in Fig.
- The downward velocity is proportional to the square of the height.
  - At higher altitudes, a large downward velocity is desired.
  - As the height (altitude) diminishes, the desired downward velocity gets smaller and smaller.
  - In the limit, as the height tends to be zero, the downward velocity also goes to zero.
- The states:
  - *h*:height above ground
  - *v*: vertical velocity of the aircraft
- The control signal
  - *f*: force



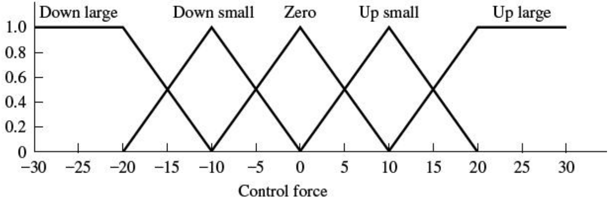
- Mass m moving with velocity v has momentum $p = mv$.
- If a force f is applied over a time interval $\Delta t \rightsquigarrow \Delta v = f\Delta t/m$
- $\Delta t = 1.0(s)$ and $m = 1.0lb \rightsquigarrow \Delta v = f$
- $\therefore v_{i+1} = v_i + f_i, \quad h_{i+1} = h_i + v_i \Delta t$
  - v_{i+1}: new velocity, v_i: old velocity
  - h_{i+1} new height, h_i old height


Near zero
Small
Medium
Large
1.0
0.8
0.6
0.4
0.2
0
0
100
200
300
400
500
600
700
800
900
1000
Height (ft)


Down large
Down small
Zero
Up small
Up large
1.0
0.8
0.6
0.4
0.2
0
−30
−25
−20
−15
−10
−5
0
5
10
15
20
25
30
Vertical velocity (ft/s)

Membership values for control force

	Output force												
	−30	**−25**	**−20**	**−15**	**−10**	**−5**	**0**	**5**	**10**	**15**	**20**	**25**	**30**
Up large (UL)	0	0	0	0	0	0	0	0	0	0.5	1	1	1
Up small (US)	0	0	0	0	0	0	0	0.5	1	0.5	0	0	0
Zero (Z)	0	0	0	0	0	0.5	1	0.5	0	0	0	0	0
Down small (DS)	0	0	0	0.5	1	0.5	0	0	0	0	0	0	0
Down large (DL)	1	1	1	0.5	0	0	0	0	0	0	0	0	0

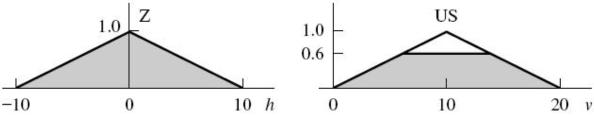


- The rules are summarized in the table

	Velocity				
Height	**DL**	**DS**	**Zero**	**US**	**UL**
L	Z	DS	DL	DL	DL
M	US	Z	DS	DL	DL
S	UL	US	Z	DS	DL
NZ	UL	UL	Z	DS	DS

- Let us use
  - singleton fuzzifier,
  - min inf. eng.
  - centroid defuzzifier

- Initial height, $h_0 : 1000ft$; Initial velocity, $v_0 : -20ft/s$
- $h = 1$ for L and 0.6 for M
- $v = 1$ for DL
- $\therefore$
  $L(1.0)$ AND $DL(1.0) \Rightarrow Z$
  $M(0.6)$ AND $DL(1.0) \Rightarrow US$
- Using defuzzifier: $f_0 = 5.8lb$



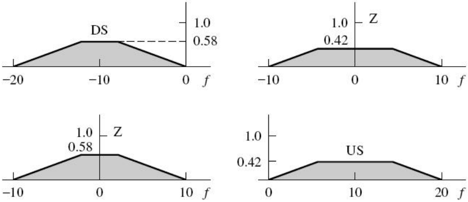
- $h_1 = h_0 + v_0 = 980ft;$
- $v_1 = v_0 + f_0 = -14.2ft/s$
- $h_1 = 0.96$ for L and 0.64 for M
- $v_1 = 0.58$ for DS and 0.42 for DL
- $\therefore$

  $L(0.96)$ AND $DS(0.58) \Rightarrow DS$

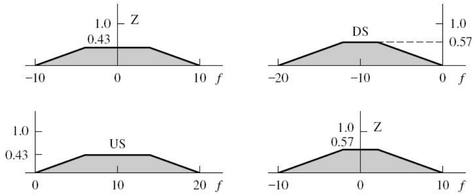
  $L(0.96)$ AND $DL(0.42) \Rightarrow Z$

  $M(0.64)$ AND $DS(0.58) \Rightarrow Z$

  $M(0.64)$ AND $DL(0.42) \Rightarrow US$
- Using defuzzifier: $f_1 = -0.5lb$



- $h_2 = h_1 + v_1 = 965.8ft;$
- $v_2 = v_1 + f_1 = -14.7ft/s$
- $h = 0.93$ for L and 0.67 for M
- $v = 0.57$ for DS and 0.43 for DL
- $\therefore$
  $L(0.93)$ AND $DL(0.43) \Rightarrow Z$
  $L(0.93)$ AND $DS(0.57) \Rightarrow DS$
  $M(0.67)$ AND $DL(0.43) \Rightarrow US$
  $M(0.67)$ AND $DS(0.57) \Rightarrow Z$
- Using defuzzifier: $f_2 = -0.4lb$

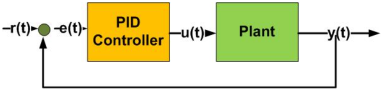


Summary of four-cycle simulation results

	Cycle 0	Cycle 1	Cycle 2	Cycle 3	Cycle 4
Height, ft	1000.0	980.0	965.8	951.1	936.0
Velocity, ft/s	−20	−14.2	−14.7	−15.1	−14.8
Control force	5.8	−0.5	−0.4	0.3	

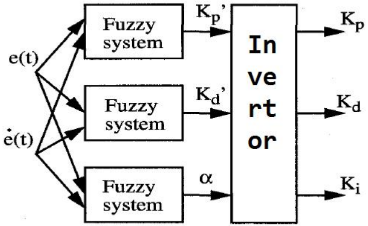
PID Controller

- The transfer function of a PID controller: $G(s) = K_p + K_i/s + K_d s$
- $\therefore u(t) = K_p[e(t) + \frac{1}{T_i}\int_0^t e(r)dr + T_d\dot{e}(t)]$
  - $T_i = K_p/K_i, \;\; T_d = K_d/K_p$
- The PID gains are usually turned by experienced human experts based on some "rule of thumb."
- By using fuzzy systems, we are trying to adjust the PID gains online

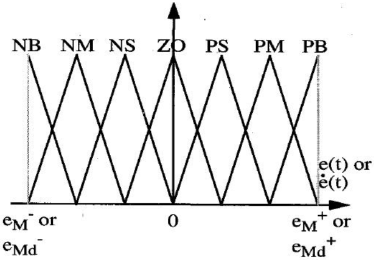


Fuzzy System for PID [1]

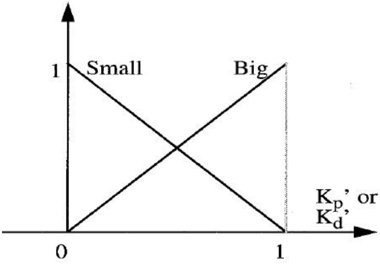
- Assume the range of PID gains:
  - $K_p \in [K_{pmax}, K_{pmin}] \subset R$
  - $K_d \in [K_{dmax}, K_{dmin}] \subset R$
- Normalize K_p and K_d to the range of $[0, 1]$
  - $K_{p'} = \frac{K_p - K_{pmin}}{K_{pmax} - K_{pmin}}$
  - $K_{d'} = \frac{K_d - K_{dmin}}{K_{dmax} - K_{dmin}}$
- Assume $T_i = \alpha T_d \rightsquigarrow K_i = \frac{K_p}{\alpha T_d} = \frac{K_p^2}{\alpha K_d}$
- Inputs of fuzzy system: $e(t), \dot{e}(t)$
- Output of fuzzy system: $K_{p'}, K_{d'}, \alpha$



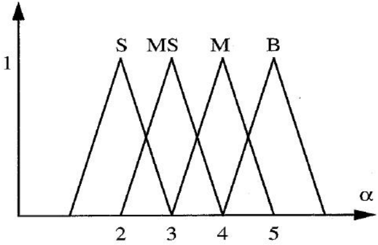
- Domain of interest:
  - $e(t) \in [e_M^-, e_M^+]$
  - $\dot{e}(t) \in [e_{Md}^-, e_{Md}^+]$
- Mem. fcn for $e(t)$ and $\dot{e}(t)$



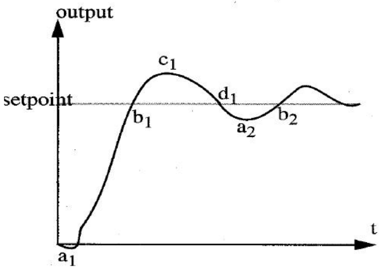
- Mem. fcn for $K_{p'}$ and $K_{d'}$



- Mem. fcn for α



- Derive the rules experimentally based on the typical step response



- For e.g., Around b_1, a small control signal requires to avoid a large overshoot ∴:
  - Small $K_{p'}$
  - Large $K_{d'}$
  - Large α
- IF $e(t)$ is ZO and $\dot{e}(t)$ is NB, THEN $K_{p'}$ is Small, $K_{d'}$ is Big, α is B
- Find the rules for other points.

- Rules for $K_{p'}$

		$\dot{e}(t)$						
		NB	NM	NS	ZO	PS	PM	PB
e(t)	NB	B	B	B	B	B	B	B
	NM	S	B	B	B	B	B	S
	NS	S	S	B	B	B	S	S
	ZO	S	S	S	B	S	S	S
	PS	S	S	B	B	B	S	S
	PM	S	B	B	B	B	B	S
	PB	B	B	B	B	B	B	B

- Rules for $K_{d'}$

		$\dot{e}(t)$						
		NB	NM	NS	ZO	PS	PM	PB
e(t)	NB	S	S	S	S	S	S	S
	NM	B	B	S	S	S	B	B
	NS	B	B	B	S	B	B	B
	ZO	B	B	B	B	B	B	B
	PS	B	B	B	S	B	B	B
	PM	B	B	S	S	S	B	B
	PB	S	S	S	S	S	S	S

- Rules for α

		$\dot{e}(t)$						
		NB	NM	NS	ZO	PS	PM	PB
e(t)	NB	2	2	2	2	2	2	2
	NM	3	3	2	2	2	3	3
	NS	4	3	3	2	3	3	4
	ZO	5	4	3	3	3	4	5
	PS	4	3	3	2	3	3	4
	PM	3	3	2	2	2	3	3
	PB	2	2	2	2	2	2	2

- There are 49 rules for each output
- Consider a fuzzy system with product inference engine, singleton fuzzifier, and center average defuzzifier

$$K_{p'} = \frac{\sum_{l=1}^{49} \bar{y}_p^l \mu_{A^l}(e(t))\mu_{B^l}(\dot{e}(t))}{\sum_{l=1}^{49} \mu_{A^l}(e(t))\mu_{B^l}(\dot{e}(t))} \quad K_{d'} = \frac{\sum_{l=1}^{49} \bar{y}_d^l \mu_{A^l}(e(t))\mu_{B^l}(\dot{e}(t))}{\sum_{l=1}^{49} \mu_{A^l}(e(t))\mu_{B^l}(\dot{e}(t))}$$

$$\alpha(t) = \frac{\sum_{l=1}^{49} \bar{y}_\alpha^l \mu_{A^l}(e(t))\mu_{B^l}(\dot{e}(t))}{\sum_{l=1}^{49} \mu_{A^l}(e(t))\mu_{B^l}(\dot{e}(t))}$$

Z.Y. Zhao, M. Tomizuka, and S. Isaka, "Fuzzy gain scheduling of pid controllers," *IEEE Trans. on Systems, Man, and Cybernetic* vol. 23, no 5 , pp. 1392–1398, 1993.