Computational Intelligence
Lecture 15: Fuzzifiers and Defuzzifiers

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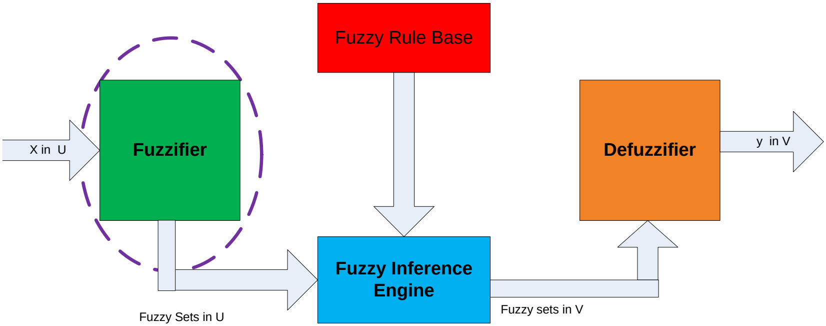
Fuzzifiers

Defuzzifier

Fuzzifiers


Fuzzy Rule Base
X in U
Fuzzifier
y in V
Defuzzifier
Fuzzy Inference Engine
Fuzzy Sets in U
Fuzzy sets in V

- Fuzzification is defined as a mapping from a real-valued point $x^* \in U \subset R^n$ to a fuzzy set $A' \in U$.
- Many crisp deterministic quantities are not deterministic in real world.
- If the form of uncertainty happens to due to imprecision, ambiguity, or vagueness, then the variable is probably fuzzy and can be represented by a membership function.
- In fuzzy control, the inputs generally originate from a piece of hardware, or a sensor
  - The measured input could be fuzzified in the rule-based system which describes the fuzzy controller.
- If the system to be controlled is not hardware based, e.g., an economic system or an ecosystem subjected to a toxic chemical, then the inputs could be scalar quantities of statistical sampling,
  - These scalar quantities could be translated into a membership function

The criteria in designing the fuzzifier

1. For crisp point x^* should have large membership value in fuzzy set A'
2. If the input of fuzzy system is corrupted by noise, then the fuzzifier should help to suppress the noise.
3. The fuzzifier should help to simplify the computations involved in the fuzzy inference engine.

Three Popular Fuzzifiers

- **Singleton fuzzifier:** $\mu_{A'}(x) = \begin{cases} 1 & if \ \ x = x^* \\ 0 & otherwise \end{cases}$
- **Gaussian fuzzifier:** $\mu_{A'}(x) = e^{-(\frac{x_1 - x_1^*}{a_1})^2} \star \ldots \star e^{-(\frac{x_n - x_n^*}{a_n})^2}$ where a_i is pos. const. and t-norm $\star$ is usually algebraic product or min
- **Triangular fuzzifier:** $\mu_{A'}(x) =$
$$\begin{cases} (1 - \frac{|x_1 - x_1^*|}{b_1}) \star \ldots \star (1 - \frac{|x_n - x_n^*|}{b_n}) & if \ |x_i - x_i^*| \geq b_i, \ \ i = 1, \ldots, n \\ 0 & otherwise \end{cases}$$
where b_i is pos. const. and t-norm $\star$ is usually algebraic product or min

- Consider the product inf. eng., where $\mu_{A_i^l} = e^{-\left(\frac{x_i - \bar{x}_i^l}{\sigma_i^l}\right)^2}$, $\bar{x}_i^l, \sigma_i^l$ conts., $i = 1, ..., n, \ \ l = 1, ..., M$
- Use Gaussian fuzzifier with algebric prod. as t-norm:

  $\mu_{B'}(y) = \max_{l=1}^{M}[\prod_{i=1}^{n} e^{-\left(\frac{x_{iP}^l - \bar{x}_i^l}{\sigma_i^l}\right)^2} e^{-\left(\frac{x_{iP}^l - \bar{x}_i^*}{a_i}\right)^2} \mu_{B^l}(y)]$

  where $x_{iP}^l = \frac{a_i^2 \bar{x}_i^l + (\sigma_i^l)^2 x_i^*}{a_i^2 + (\sigma_i^l)^2}$
- Assume x_i^* is corrupted by noise: $x_i^* = x_{i-}^* + n_i^*$, $(n_i$: noise$)$
  - $\therefore \ x_{iP}^l = \frac{a_i^2 \bar{x}_i^l + (\sigma_i^l)^2 x_{i0}^*}{a_i^2 + (\sigma_i^l)^2} + \frac{(\sigma_i^l)^2}{a_i^2 + (\sigma_i^l)^2} n_i^*$
  - the noise is suppressed by $\frac{(\sigma_i^l)^2}{a_i^2 + (\sigma_i^l)^2} n_i^*$
  - The larger a_i than σ_i^l, the more suppressed n_i^*
- Exercise: Show that the triaggular fuzzifier has the same capability.

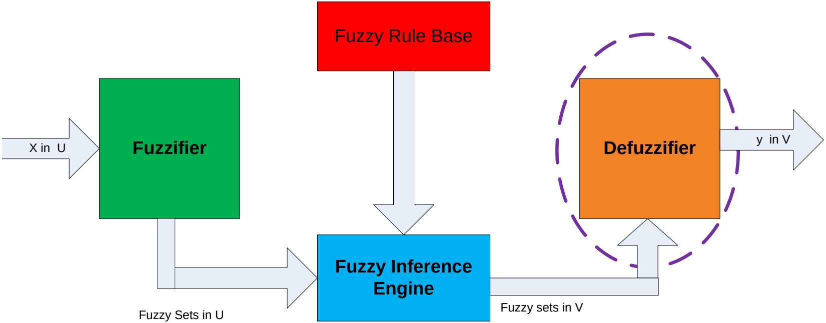
- Consider the min inf. eng., where $\mu_{A_i^l} = e^{-(\frac{x_i - \bar{x}_i^l}{\sigma_i^l})^2}$, $\bar{x}_i^l, \sigma_i^l$ conts., $i = 1, ..., n,\ \ l = 1, ..., M$
- Use Gaussian fuzzifier with min as t-norm:

$$\mu_{B'}(y) = \max_{l=1}^{M}[\min(e^{-(\frac{x_{iM}^l - \bar{x}_i^l}{\sigma_i^l})^2}, \ldots, e^{-(\frac{x_{nM}^l - \bar{x}_n^*}{\sigma_n^l})^2} \mu_{B^l}(y))]$$

where $x_{iM}^l = \frac{a_i^2 \bar{x}_i^l + \sigma_i^l x_i^*}{a_i^2 + \sigma_i^l}$

- As a conclusion
  - The singleton fuzzifier simplifies the computation involved in the fuzzy inference engine for any type of membership functions
  - The Gaussian / triangular fuzzifiers also simplify the computation in the fuzzy inference engine, if the membership functions in the IF-THEN rules are Gaussian/triangular.
  - The Gaussian and triangular fuzzifiers can suppress noise in the input, but the singleton fuzzifier cannot.

Defuzzifiers



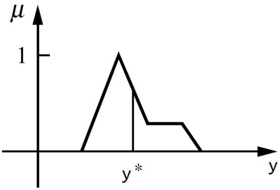
- Defuzzification a mapping from fuzzy set $B' \in V \subset R$ (the output of the fuzzy inference engine) to crisp point $y^* \in V$.
- It is specifying a point in V as best representative of the fuzzy set B'.
- For example, in classification and pattern recognition a fuzzy partition or pattern should be transformed to a crisp partition or pattern
- In control a single-valued input should be given to a semiconductor device instead of a fuzzy input command.

The criteria in designing the defuzzifier

1. Plausibility: The point y^* should represent B' from an intuitive point of view; e.g., it may lie approximately in the middle or hight of the support of B'
2. Computational simplicity: It is important for real-time systems such as fuzzy control.
3. Continuity: A small change in B' should not result in a large change in y^*.

Some Popular Defuzzifiers

1. Centriod method or Center of Gravity: [1], [2]: $y^* = \frac{\int_V y\mu_{B'}(y)dy}{\int_V \mu_{B'}(y)dy}$

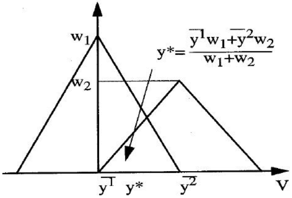


where $\int_V$ is the conventional integral.

- Assuming $\mu'_B(y)$ as the probability density function of a random variable, ⇝ centriod defuzzifier gives the mean value of the random variable
- Some times, it is desired to eliminate too small membership fcn. of B' ⇝ define α-cut set $V_\alpha = \{y \in V | \mu'_B(y) \geq \alpha\}$
  $y^* = \frac{\int_{V_\alpha} y\mu_{B'}(y)dy}{\int_{V_\alpha} \mu_{B'}(y)dy}$
- Advantages: It is intuitively plausible
- Disadvantages: It is computationally intensive. specifically when $\mu_{B'}$ is irregular.

Some Popular Defuzzifiers

2. Center Average Defuzzifier or Weighted average method Considering B' is the union or intersection of M fuzzy sets $y^* = \frac{\sum_{l=1}^{M} \bar{y}^l \mu(\bar{y}^l)}{\sum_{l=1}^{M} \mu(\bar{y}^l)}$

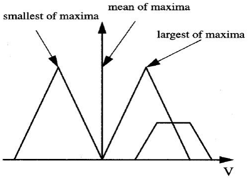


where $\bar{y}^l$ is center of lth fuzzy set

- It is the most popular diffuzifier in fuzzy systems and control
- Advantages: It is intuitively plausible and computationally simple; small changes in $\bar{y}^l$ and $\mu(\bar{y}^l)$ result in small changes in y^*
- Disadvantages: It is usually acceptable for symmetric output membership functions

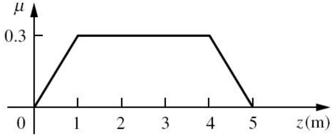
3. Maximum Defuzzifier: Choose the point in V corresponding to the height of $\mu_{B'}(y)$:
$hgt(B') = \{y \in V | \mu_{B'}(y) = \sup_{y \in V} \mu_{B'}(y)\}$
$hgt(B')$: can be a set of points with max memb. fcn.

- If $hgt(B')$ is a single point $\rightsquigarrow y^* = hgt(B')$
- Otherwise one of the following definitions can be used:
  - smallest of maxima defuzzifier: $y^* = \inf\{y \in hgt(B')\}$
  - largest of maxima defuzzifier: $y^* = \sup\{y \in hgt(B')\}$
  - mean of maxima defuzzifier: $y^* = \frac{\int_{hgt(B')} ydy}{\int_{hgt(B')} dy}$ ($\int_{hgt}$ is usual integration if $hgt(B')$ is continuous, it is summation if $hgt(B')$ is discrete)
- Advantages: intuitively plausible, computationally simple
- Disadvantages: small changes in B' may result in large changes in y^* (if the memb. fcn $\mu_{B'}$ is nonconvex)

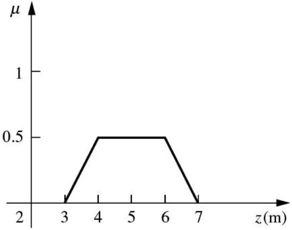


Example

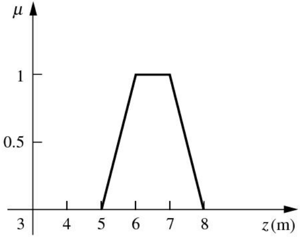
- A railroad company intends to lay a new rail line in a particular part of a county.
- The area that the new line is passing, must be purchased for right-of-way considerations.
- There are three surveys in right-of-way widths, in meters
- But the info are not precise since
  - some of the land along the proposed railway route is already public domain and will not need to be purchased.
  - The original surveys are so old that some ambiguity exists on boundaries and public right-of-way for old utility lines and old roads.
- The three fuzzy sets B_1, B_2, B_3 represent the uncertainty in each survey as to the membership of right-of-way width, in meters, in privately owned land.
- We need to find the single most nearly representative right-of-way width (z) to purchase



B_1



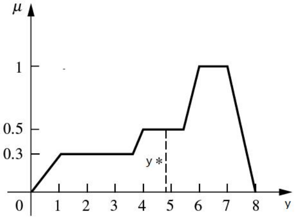
B_2



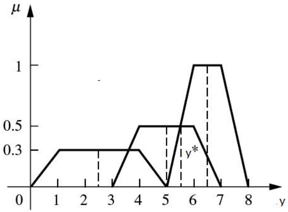
B_3

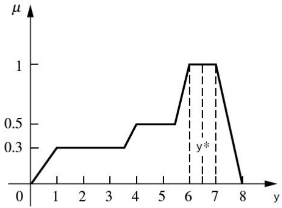
Example Cont'd: Centriod method:

- $y^* = [\int_0^1 (0.3y)ydy + \int_1^{3.6} 0.3ydy + \int_{3.6}^4 (\frac{y-3.6}{2})ydy + \int_4^{5.5} 0.5ydy + \int_{5.5}^6 (y-5.5)ydy + \int_6^7 ydy + \int_7^8 (\frac{7-y}{2})ydy] \div [\int_0^1 (0.3y)dy + \int_1^{3.6} 0.3dy + \int_{3.6}^4 (\frac{y-3.6}{2})dy + \int_4^{5.5} 0.5dy + \int_{5.5}^6 (y-5.5)dy + \int_6^7 dy + \int_7^8 (\frac{7-y}{2})dy] = 4.9m$



- Weighted Average method: $y^* = \frac{(0.3\times2.5)+(0.5\times5)+(1\times6.5)}{0.3+0.5+1} = 5.41m$





- Mean max method: $y^* = \frac{6+7}{2} = 6.5m$

Comparison of Defuzzifiers

	center of gravity	center average	maximum
plausibility	yes	yes	yes
computational simplicity	no	yes	yes
continuity	yes	yes	no

- C. Lee, "Fuzzy logic in control systems: fuzzy logic controller, parts i and ii," *Fuzzy logic in control systems: fuzzy logic controller, Parts I and II*, vol. 20 , pp. 404–435, 1990.
- M. Sugeno, "An introductory survey of fuzzy control," *Inf. Sci* vol. 36 , pp. 59–83, 1985.