Computational Intelligence
Lecture 13: Simple Neural Networks for Pattern Classification

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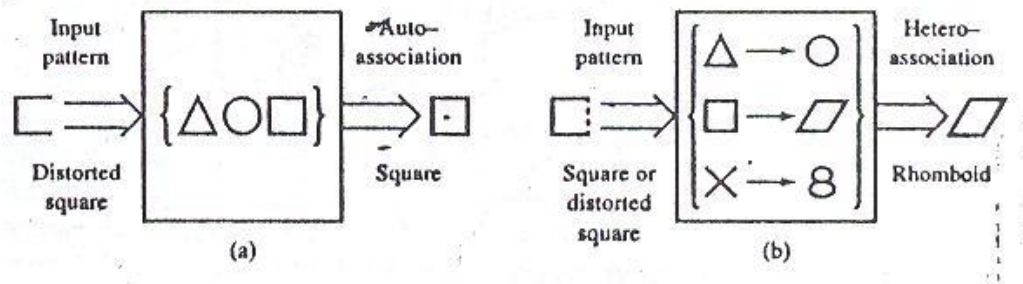
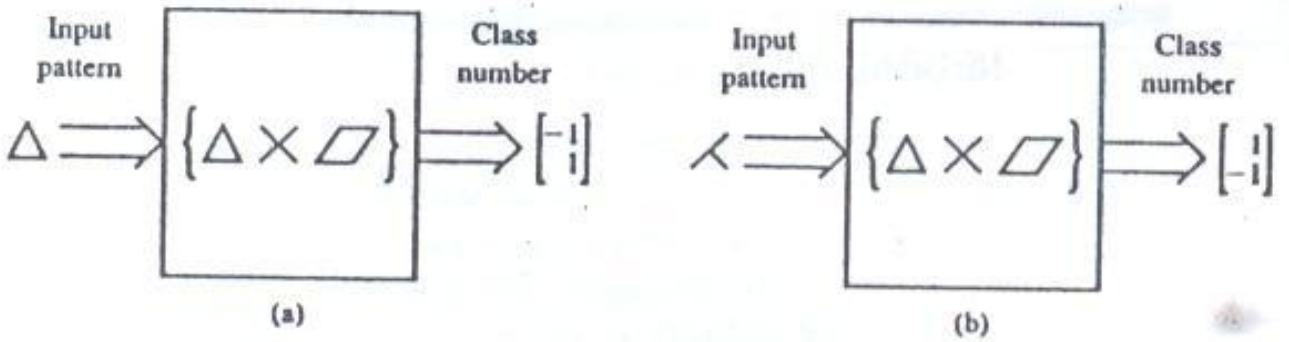
Neural Processing

- One of the most applications of NN is in mapping inputs to the corresponding outputs $o = f(wx)$
- The process of finding o for a given x is named **recall**.
- Assume that a set of patterns can be stored in the network.
- **Autoassociation:** The network presented with a pattern similar to a member of the stored set, it associates the input with the closest stored pattern.
  - A degraded input pattern serves as a cue for retrieval of its original
- **Hetroassociation:** The associations between pairs of patterns are stored.



Association response: (a) autoassociation and (b) heteroassociation.

- **Classification:** The set of input patterns is divided into a number of classes. The classifier recall the information regarding class membership of the input pattern. The outputs are usually binary.
  - Classification can be considered as a special class of hetroassociation.
- *Recognition:* If the desired response is numbers but input pattern does not fit any pattern.

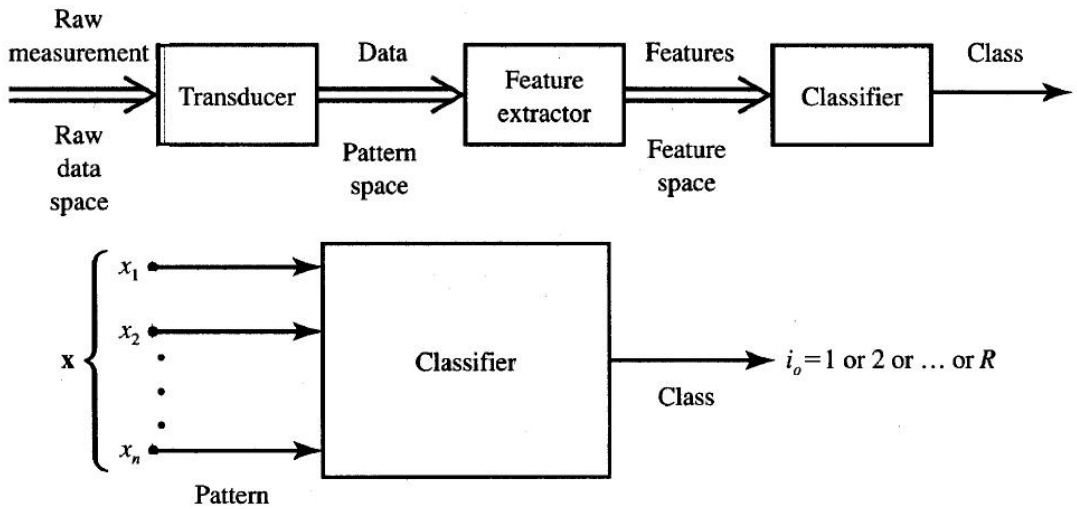


Classification response: (a) classification and (b) recognition.

- **Function Approximation:** Having I/O of a system, their corresponding function f is approximated.
  - This application is useful for control
- In all mentioned aspects of neural processing, it is assumed the data is already stored to be recalled
- Data are stored in a network in learning process

Pattern Classification

- The goal of pattern classification is to assign a physical object, event, or phenomenon to one of predefined classes.
- Pattern is quantified description of the physical object or event.
- Pattern can be based on time (sensors output signals, acoustic signals) or place (pictures, fingertips):
- Example of classifiers: disease diagnosis, fingertip identification, radar and signal detection, speech recognition
- Fig. shows the block diagram of pattern recognition and classification

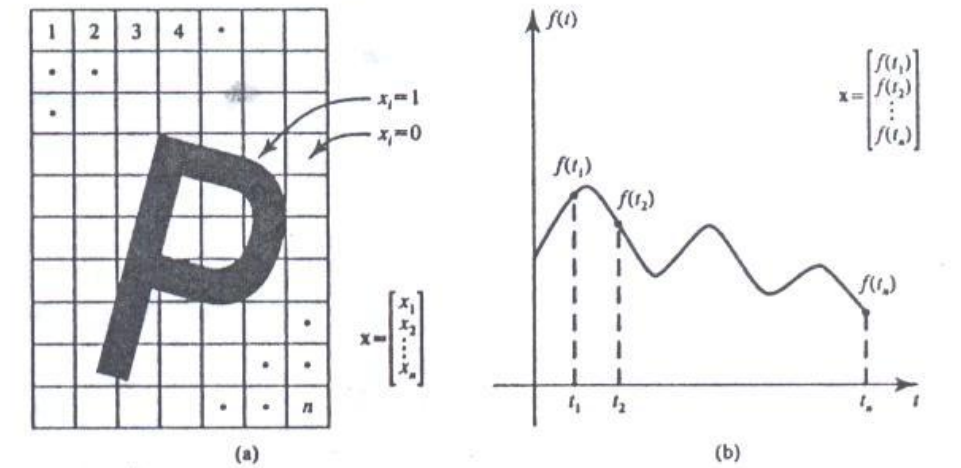


Pattern Classification

- Input of feature extractors are sets of data vectors belonging to a certain category.
- Feature extractor compress the dimensionality as much as does not ruin the probability of correct classification
- Any pattern is represented as a point in n-dimensional Euclidean space E^n, called pattern space.
- The points in the space are n-tuple vectors $X = [x_1 \ ... \ x_n]^T$.
- A pattern classifier maps sets of points in E^n space into one of the numbers $i_0 = 1, ..., R$ based on decision function $i_0 = i_0(x)$.
- The set containing patterns of classes 1,..., R are denoted by $\S_i, ...\S_n$

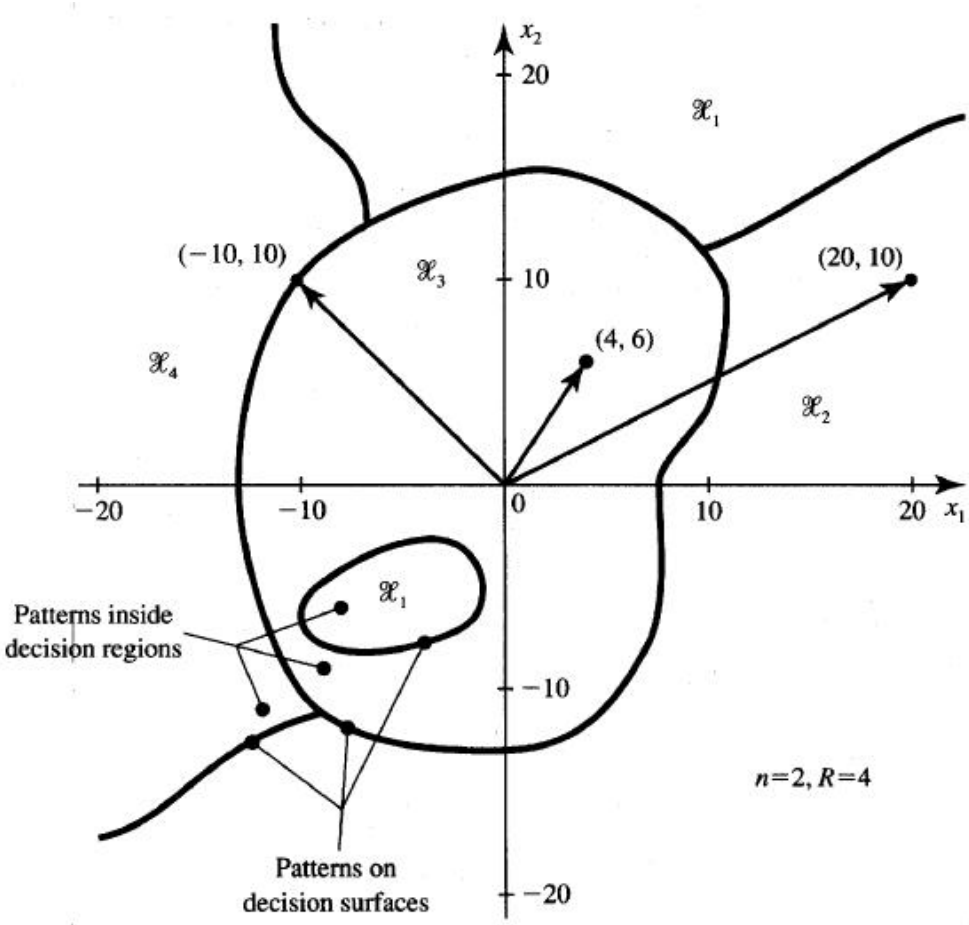
Pattern Classification

- The fig. depicts two simple method to generate the pattern vector
  - Fig. a: x_i of vector $X = [x_1 \ ... \ x_n]^T$ is 1 if ith cell contains a portion of a spatial object, otherwise is 0
  - Fig b: when the object is continuous function of time, the pattern vector is obtained at discrete time instance t_i, by letting $x_i = f(t_i)$ for $i = 1, ..., n$



Two simple ways of coding patterns into pattern vectors: (a) spatial object and (b) temporal object (waveform).

- Example: for $n = 2$ and $R = 4$
- $X = [20\ 10] \in \S_2$, $X = [4\ 6] \in \S_3$
- The regions denoted by $\S_i$ are called decision regions.
- Regions are seperated by decision surface
  - The patterns on decision surface does not belong to any class
  - decision surface inE^2 is curve, for general case, E^n is $(n-1)-$dimentional hepersurface.



Discriminant Functions

- During the classification, the membership in a category should be determined by classifier based on discriminant functions $g_1(X), ..., g_R(X)$
- Assume $g_i(X)$ is scalar.
- The pattern belongs to the ith category iff $g_i(X) > g_j(X)$ for $i, j = 1, ..., R, i \neq j$.
- $\therefore$ within the region $\S_i$, ith discriminant function have the largest value.
- Decision surface contain patterns X without membership in any classes
- The decision surface is defined as:

$$g_i(X) - g_j(X) = 0$$

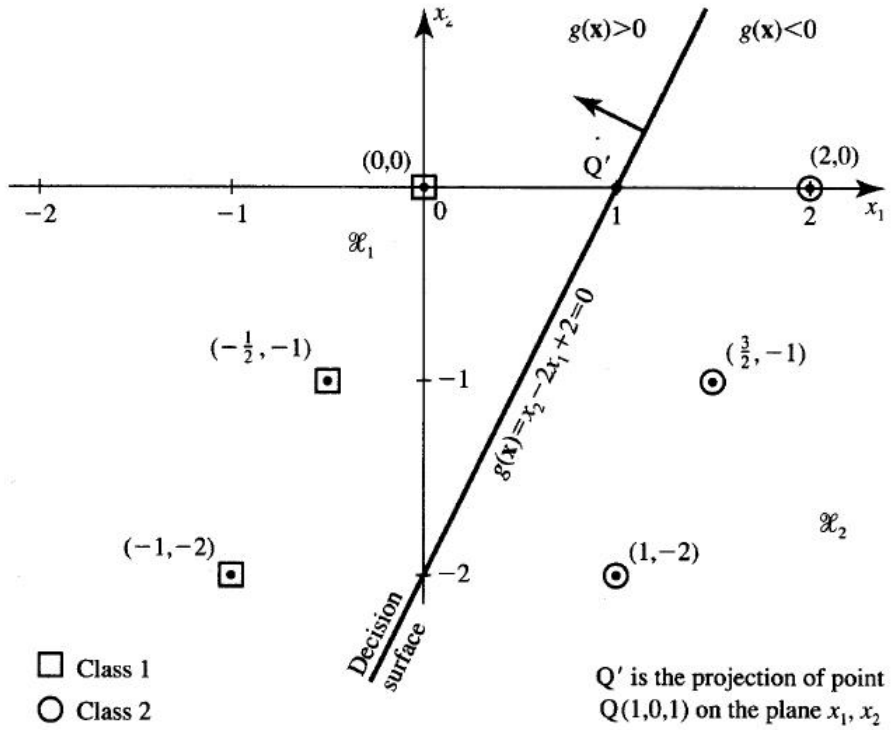
- **Example:** Consider six patterns, in two dimensional pattern space to be classified in two classes:
  $\{[0\ 0]', [-0.5\ -1]', [-1\ -2]'\}$: class 1
  $\{[2\ 0]', [1.5\ -1]', [1\ -2]'\}$: class 2
- Inspection of the patterns shows that the $g(X)$ can be arbitrarily chosen

$$g(X) = -2x_1 + x_2 + 2$$

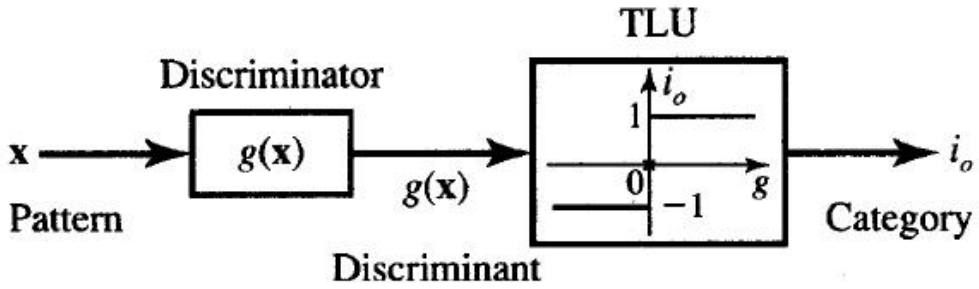
$$g(X) > 0 : \text{class 1}$$

$$g(X) < 0 : \text{class 2}$$

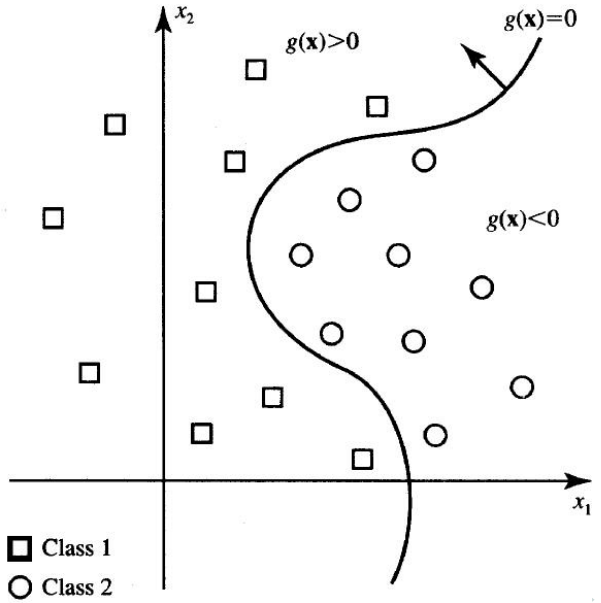
$$g(X) = 0 : \text{on the surface}$$



- The classifier can be implemented as shown in Fig. below (TLU is threshold logic



- **Example 2:** Consider a classification problem as shown in fig. below
- the discriminant surface can not be estimated easily.
- It may result in a nonlinear function of x_1 and x_2.



Pattern Classification

- In pattern classification we assume
  - The sets of classes and their members are known
- Having the patterns, We are looking to find the discriminant surface by using NN,
- The only condition is that the patterns are separable
- The patterns like first example are linearly separable and in second example are nonlinearly separable
- In first step, simple sparable systems are considered.

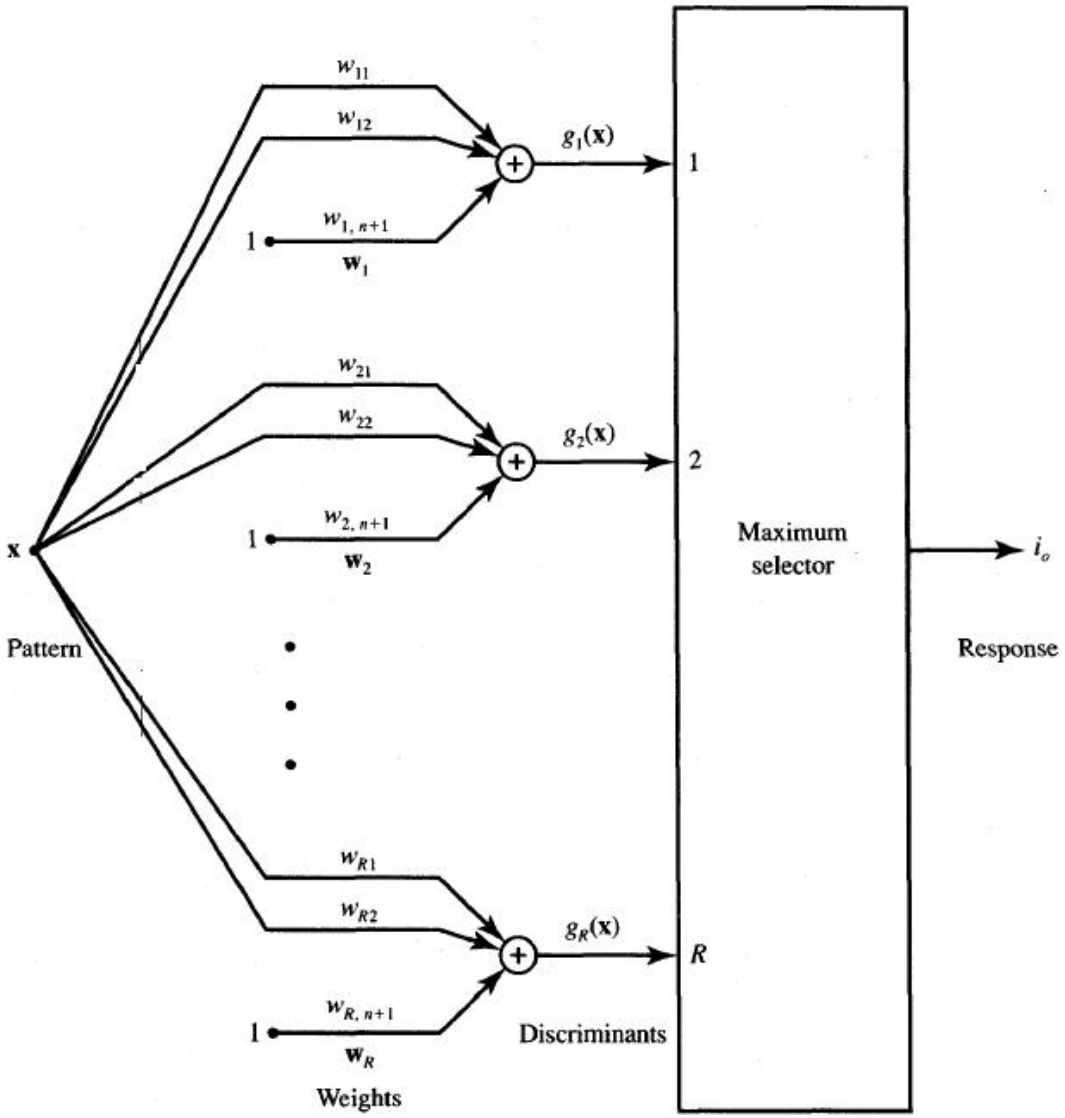
Linear Machine

- Linear discernment functions are the simplest discriminant functions:
$$g(x) = w_1x_1 + w_2x_2 + ... + w_nx_n + w_{n+1} \quad (1)$$
- Consider $\mathbf{x} = [x_1, ..., x_n]^T$, and $\mathbf{w} = [w_1, ..., w_n]^T$, (1) can be redefined as $g(x) = \mathbf{w}^T\mathbf{x} + w_{n+1}$
- Now we are looking for $\mathbf{w}$ and w_{n+1} for classification
- The classifier using the discriminant function (1) is called Linear Machine.
- Minimum distance classifier (MDC) or nearest neighborhood are employed to classify the patterns and find w's:
  - E^n is the n-dimensional Euclidean pattern space $\rightsquigarrow$ Euclidean distance between two point are $\|x_i - x_j\| = [(x_i - x_j)^T(x_i - x_j)]^{1/2}$.
  - P_i is center of gravity of cluster i.
  - A MDC computes the distance from pattern x of unknown to each prototype ($\|x - p_i\|$).
  - The prototype with smallest distance is assigned to the pattern.

Linear Machine

- Calculating the squared distance:
  $\|x - p_i\|^2 = (x - p_i)^T(x - p_i) = x^T x - 2x^T p_i + p_i^T p_i$
- $x^T x$ is independent of i
- $\therefore$ min the equation above is obtained by max the discernment function: $g_i(x) = x^T p_i - \frac{1}{2} p_i^T p_i$
- We had $g_i(x) = \mathbf{w}_i^T \mathbf{x} + w_{in+1}$
- $\therefore$ Considering $p_i = (p_{i1}, p_{i2}, ..., p_{in})^T$,
- The weights are defined as

$$\begin{aligned} w_{ij} &= p_{ij} \\ w_{in+1} &= -\frac{1}{2} p_i^T p_i, \\ & \quad i = 1, ..., R, j = 1, ..., n \end{aligned} \tag{2}$$



A linear classifier.

Example

- In this example a linear classifier is designed
- Center of gravity of the prototypes are known a priori

$$p_1 = \begin{bmatrix} 10 \\ 2 \end{bmatrix}, p_2 = \begin{bmatrix} 2 \\ -5 \end{bmatrix}, p_3 = \begin{bmatrix} -5 \\ 5 \end{bmatrix}$$

- Using (2) for $R = 3$, the weights are

$$w_1 = \begin{bmatrix} 10 \\ 2 \\ -52 \end{bmatrix}, w_2 = \begin{bmatrix} 2 \\ -5 \\ -14.5 \end{bmatrix}, w_3 = \begin{bmatrix} -5 \\ 5 \\ -25 \end{bmatrix}$$

- Discriminant functions are:

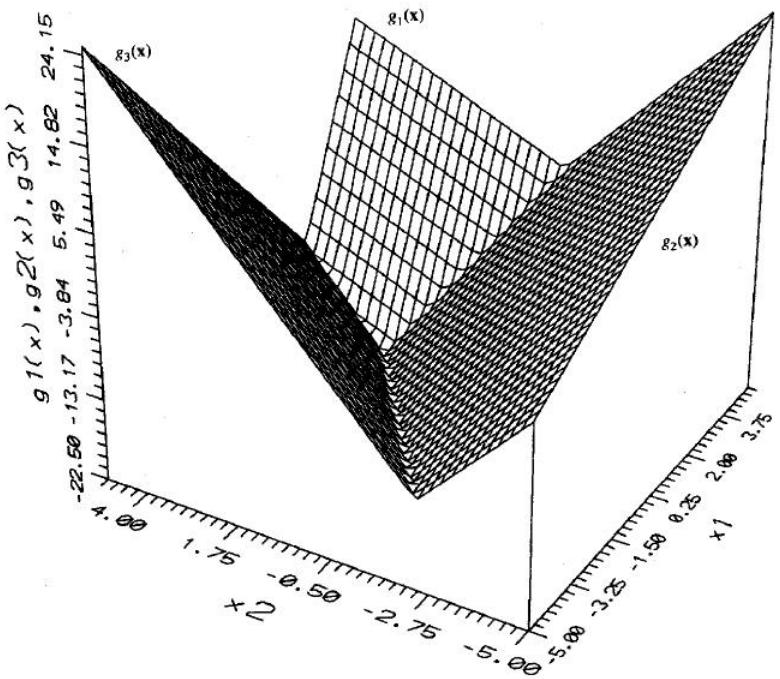
$$\begin{aligned} g_1(x) &= 10x_1 + 2x_2 - 52 \\ g_2(x) &= 2x_1 - 5x_2 - 14.5 \\ g_3(x) &= -5x_1 + 5x_2 - 25 \end{aligned}$$

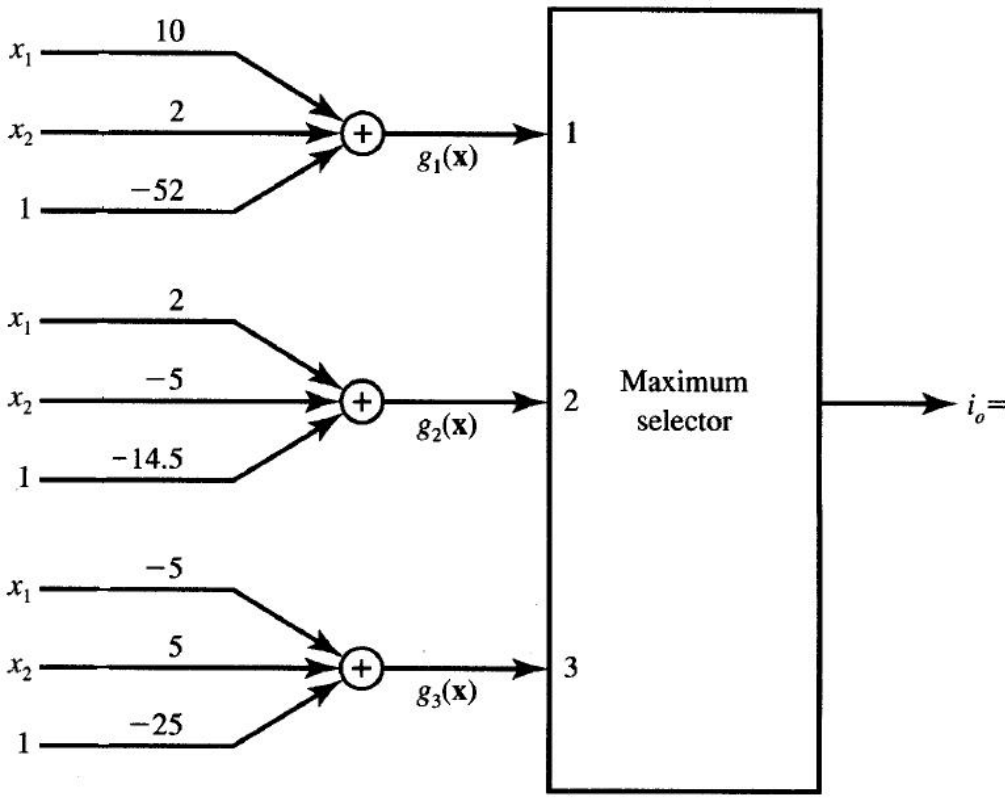
The decision lines are:

$$S_{12} : 8x_1 + 7x_2 - 37.5 = 0$$

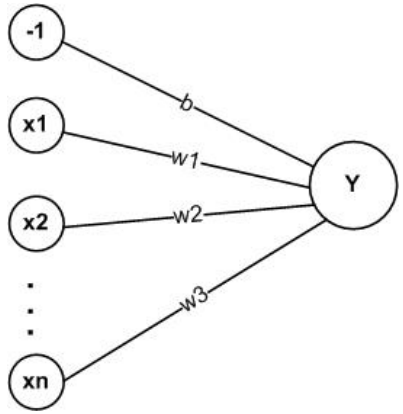
$$S_{13} : -15x_1 + 3x_2 + 27 = 0$$

$$S_{23} : -7x_1 + 10x_2 - 10.5 = 0$$





- The structure of a single layer network



- Considering the threshold (θ) the activation function is defined as

$$y = \begin{cases} 1 & net \geq \theta \\ -1 & net < \theta \end{cases} \tag{3}$$

- Bias can be considered as a weight with input 1
- The activation function can be considered as

$$y = \begin{cases} 1 & net \geq 0 \\ -1 & net < 0 \end{cases}, net = -b + \sum_{i=1}^{n} w_i x_i \tag{4}$$

- Bias can be play as a threshold in activation function.
- Using (4):

$$b + w_1 x_1 + w_2 x_2 = 0 \rightsquigarrow x_2 = -\frac{w_1}{w_2} x_1 + \frac{b}{w_2}, w_2 \neq 0$$

- Considering threshold (3):

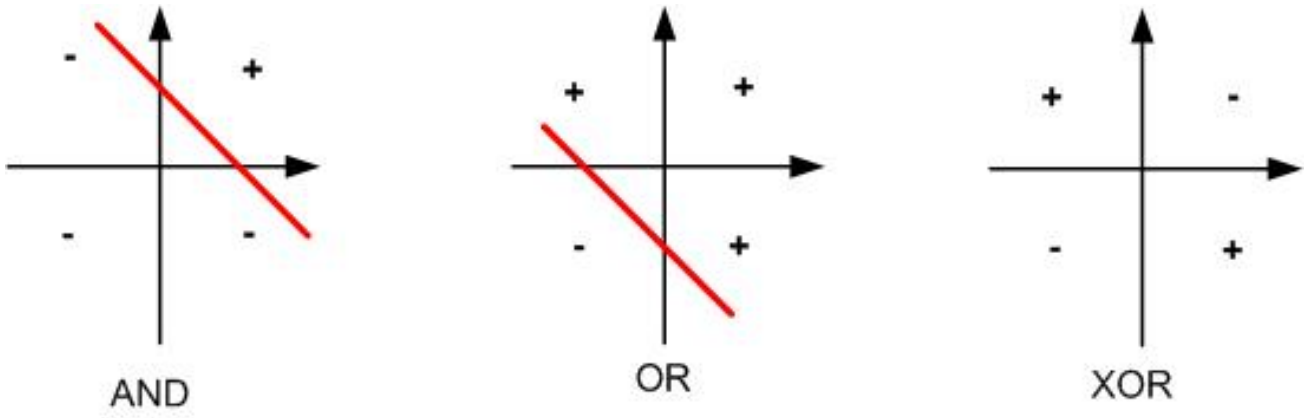
$$w_1 x_1 + w_2 x_2 = \theta \rightsquigarrow x_2 = -\frac{w_1}{w_2} x_1 + \frac{\theta}{w_2}, w_2 \neq 0$$

- $\therefore$ Both case are similar
- Considering neither threshold nor bias implies that discriminant function always intersects the origin which is not always correct.

- If R linear functions
  $g_i(x) = w_1x_1 + w_2x_2 + ... + w_nx_n + w_{n+1}, i = 1, ..., R$ exists s.t

$$g_i(x) > g_j(x) \ \forall x \in \S_i, i, j = 1, ..., R, i \neq j$$

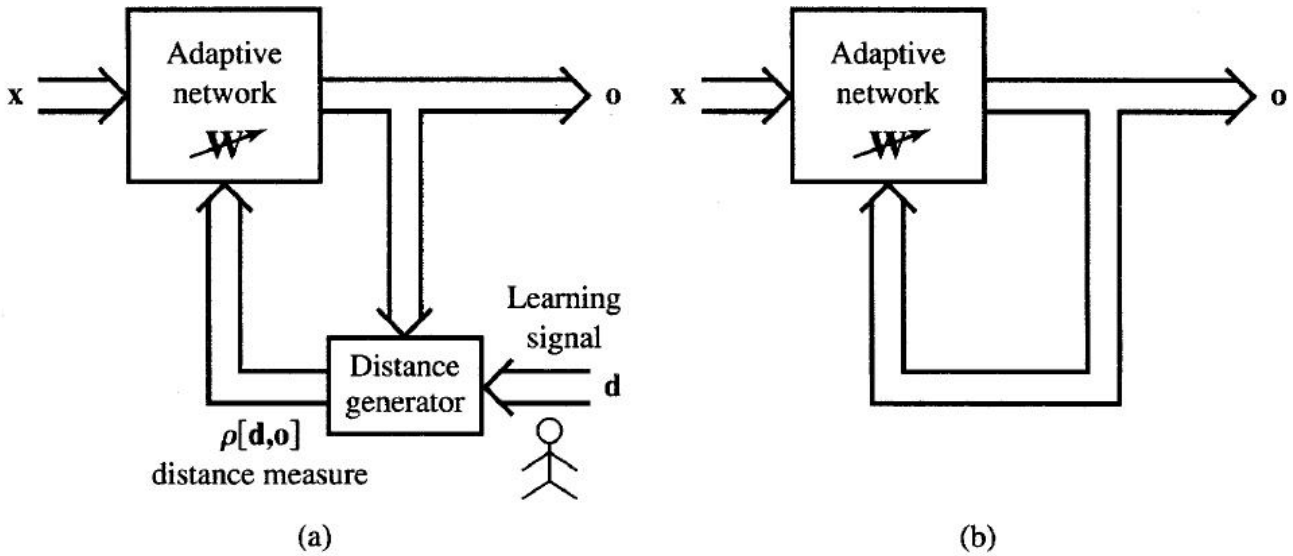
  the pattern set is linearly separable
- Single layer networks can only classify linearly separable
- Nonlinearly separable patterns are classified by multiple layer networks
- **Example:** AND: $x_2 = -x_1 + 1 \quad (b = -1, w_1 = 1, \ w_2 = 1)$
- **Example:** OR: $x_2 = -x_1 - 1 \quad (b = 1, w_1 = 1, \ w_2 = 1)$



Learning

- When there is no a priori knowledge of p_i's, a method should be found to adjust weights (w_i).
- We should learn the network to behave as we wish
- Learning task is finding w based on the set of training examples x to provide the best possible approximation of $h(x)$.
  - In classification problem $h(x)$ is discriminant function $g(x)$.
- Two types of learning is defined
  1. Supervised learning: At each instant of time when the input is applied the desired response d is provided by teacher
     - The error between actual and desired response is used to correct and adjust the weights.
     - It rewards accurate classifications/ associations and punishes those yields inaccurate response.

2. Unsupervised learning: desired response is not known to improve the network behavior.
   - A proper self-adoption mechanism have to be embedded.



Block diagram for explanation of basic learning modes: (a) supervised learning and (b) unsupervised learning.

- General learning rule is the weight vector $w_i = [w_{i1}\ w_{i2}\ \ldots\ w_{in}]^T$ increases in proportion to the product of input x and learning signal r

$$\begin{aligned} w_i^{k+1} &= w_i^k + \Delta w_i^k \\ \Delta w_i^k &= c r^k(w_i^k, x^k) x^k \end{aligned}$$

c is pos. const.: learning constant.

- For supervised learning $r = r(w_i, x, d_i)$
- For continuous learning

$$\frac{dw_i}{dt} = crx$$

Perceptron Learning Rule

- Perceptron learning rule was first time proposed by Rosenblatt in 1960.
- Learning is supervised.
- The weights are updated based the error between the system output and desired output

$$\begin{aligned} r^k &= d^k - o^k \\ \Delta W_i^k &= c(d_i^k - o_i^k)x^k \end{aligned} \tag{5}$$

- Based on this rule weights are adjusted iff output o_i^k is incorrect.
- The learning is repeated until the output error is zero for every training pattern
- It is proven that by using Perceptron rule the network can learn what it can present
  - If there are desired weights to solve the problem, the network weights converge to them.

- In classification, consider $R = 2$
  - The Decision lines are

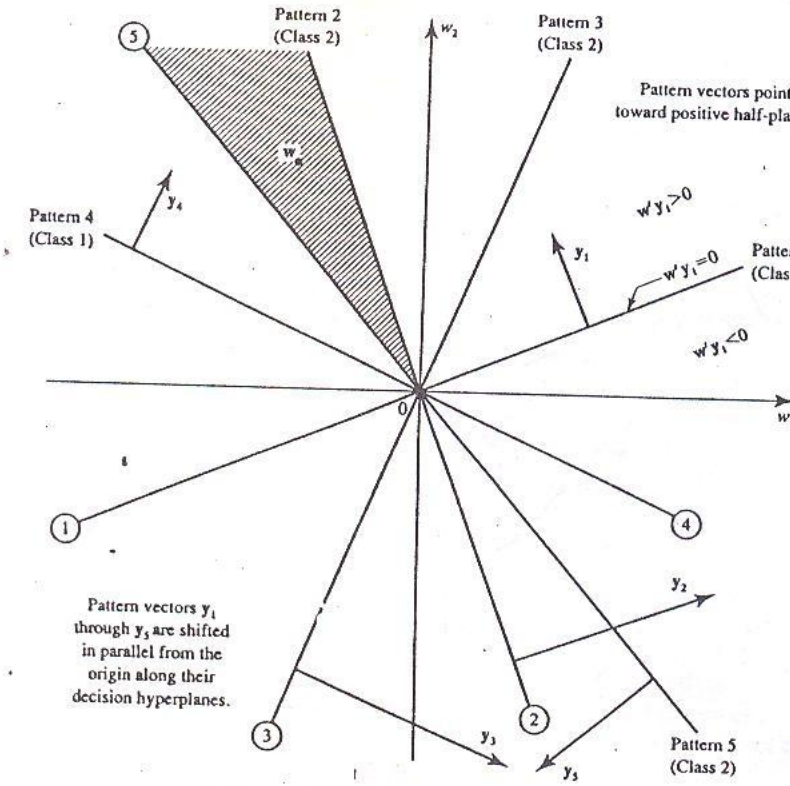
$$\begin{aligned} W^T x &- b = 0 \\ W &= [w_1,\ w_2,\ ...,\ w_n]^T, x = [x_1,\ x_2, ..., x_n]^T \end{aligned} \tag{6}$$

  - Considering bias, the augmented weight and input vectors are $y = [x_1,\ x_2, ..., x_n,\ -1]^T,\ W = [w_1,\ w_2,\ ...,\ w_n,\ b]^T$
  - Therefore, (6) can be written

$$W^T y = 0 \tag{7}$$

  - Since in learning, we are focusing on updating weights, the decision lines are presented on weights plane.
  - $\therefore$ The decision line always intersects the origin ($w = 0$).
  - Its normal vector is y which is perpendicular to the plane

- The normal vector always points toward $W^T y > 0$.
- Positive decision region, $W^T y > 0$ is class 1
- Negative decision region, $W^T y < 0$ is class 2
- Using geometrical analysis, we are looking for a guideline for developing weight vector adjustment procedure.



Decision hyperplane in augmented weight space for a five pattern set from two classes.

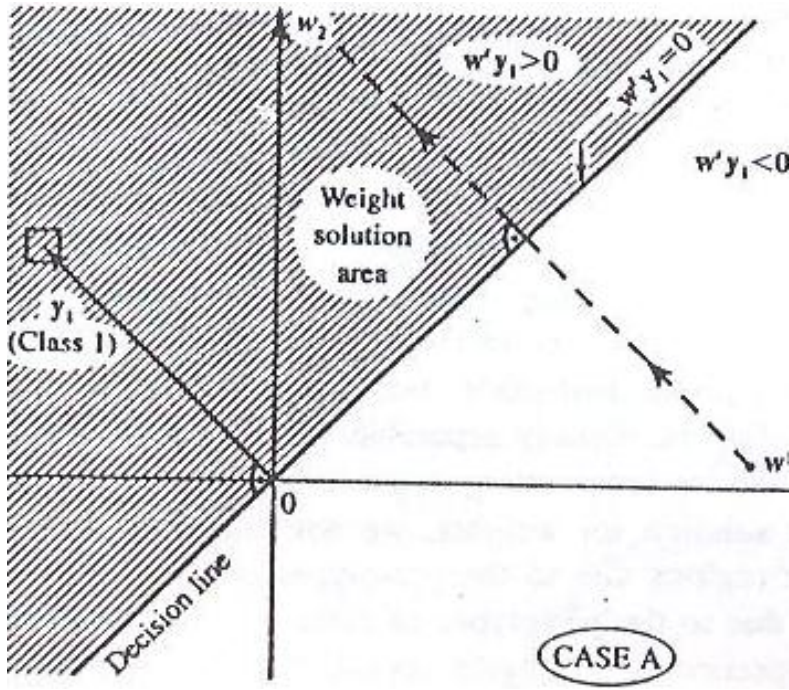
- **Case A:** W^1 is in neg. half-plane, y_1 belongs to class 1
  - $W^1 y_1 < 0 \rightsquigarrow W^T$ should be moved toward gray section.
  - The fast solution is moving W^1 in the direction of steepest increase which is the gradient

$$\nabla_w (W^T y_1) = y_1$$

  - $\therefore$ The adjusted weights become

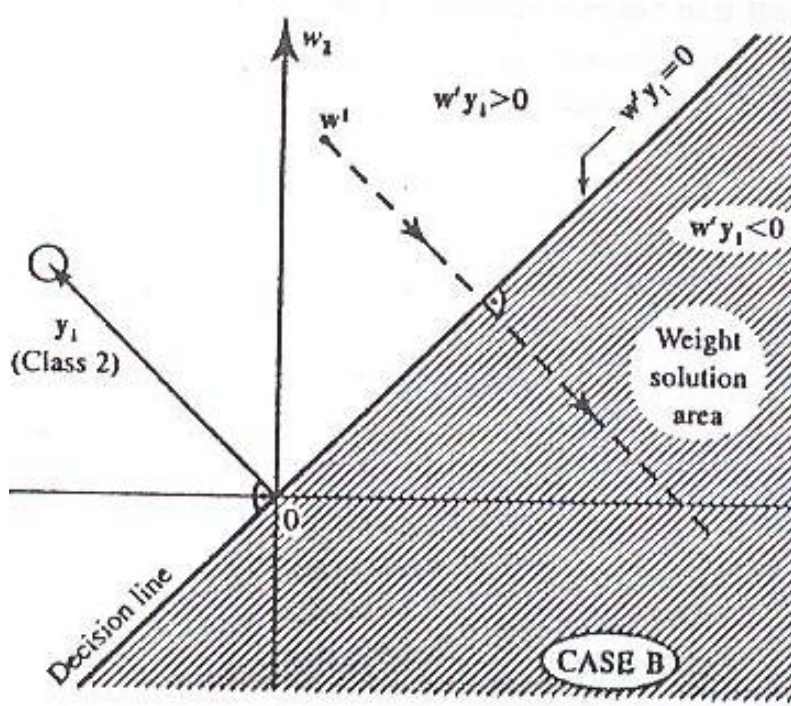
$$W' = W^1 + c y_1$$

  - $c > 0$ is called correction increment or learning rate, it is the size of adjustment.
  - W' is the weights after correction.

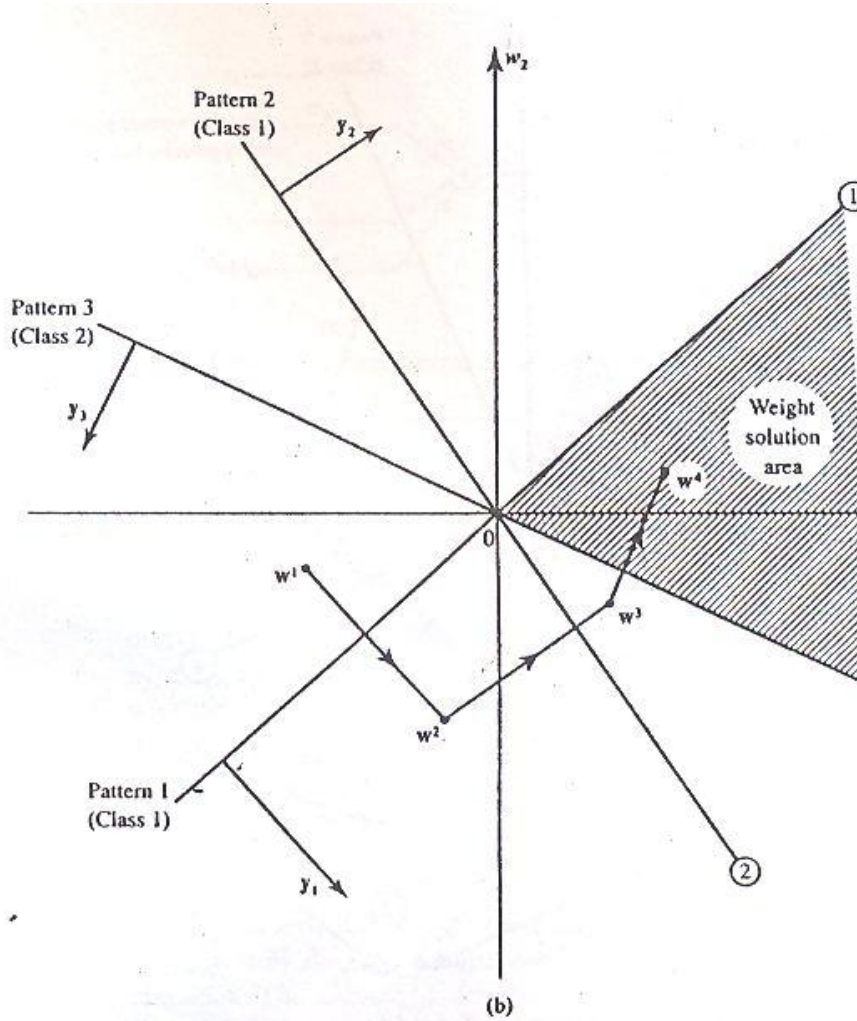


- **Case B:** W^1 is in pos. half-plane, y_1 belongs to class 2
  - $W^1 y_1 > 0 \rightsquigarrow W^T$ should be moved toward gray section.
  - The fast solution is moving W^1 in the direction of steepest decrease which is the neg. gradient
  - $\therefore$ The adjusted weights become

$$W' = W^1 - cy_1$$



- **Case C:** Consider three augmented patterns y_1, y_2 and y_3, given in sequence
- The response to y_1, y_2 should be 1 and for y_3 should be -1
- The lines 1, 2, and 3 are fixed for variable weights.
  - Starting with W^1 and input y_1, $W^T y_1 < 0 \rightsquigarrow W^2 = W^1 + cy_1$
  - For y_2, $W^T y_2 < 0 \rightsquigarrow W^3 = W^2 + cy_2$
  - For y_3, $W^T y_3 > 0 \rightsquigarrow W^4 = W^3 - cy_3$
  - W^4 is in the gray area (solution)



Weight adjustments of learning dichotomizer *(continued)*:

- To summarize, the updating rule is

$$W' = W^1 \pm cy$$

where

  - Pos. sign is for undetected pattern of class 1
  - Neg. sign is for undetected pattern of class 2
  - For correct classification, no adjustment is made.

- The updating rule is, indeed, (5)

Single Discrete Perceptron Training Algorithm

- Given P training pairs $\{x_1, d_1,\ x_2, d_2,\ ..., x_p, d_p\}$ where x_i is $(n \times 1),\ d_i$ is $(1 \times 1),\ i = 1, ..., P$
- The augmented input vectors are $y_i = \begin{bmatrix} x_i \\ -1 \end{bmatrix}$, for $i = 1, ..., P$
- In the following, k is training step and p is step counter within training cycle.
  1. Choose $c > 0$
  2. Initialized weights at small random values, w is $(n+1) \times 1$
  3. Initialize counters and error: $k \longleftarrow 1,\ p \longleftarrow 1, E \longleftarrow 0$
  4. Training cycle begins here. Set $y \longleftarrow y_p, d \longleftarrow d_p,\ o = sgn(w^T y)$ (sgn is sign function)
  5. Update weights $w \longleftarrow w + \frac{1}{2}c(d - o)y$
  6. Find error: $E \longleftarrow \frac{1}{2}(d - o)^2 + E$
  7. If $p < P$ then $p \longleftarrow p + 1, k \longleftarrow k + 1$, go to step 4, otherwise, go to step 8.
  8. If $E = 0$ the training is terminated, otherwise $E \longleftarrow 0, p \longleftarrow 1$ go to step 4 for new training cycle

Convergence of Perceptron Learning Rule

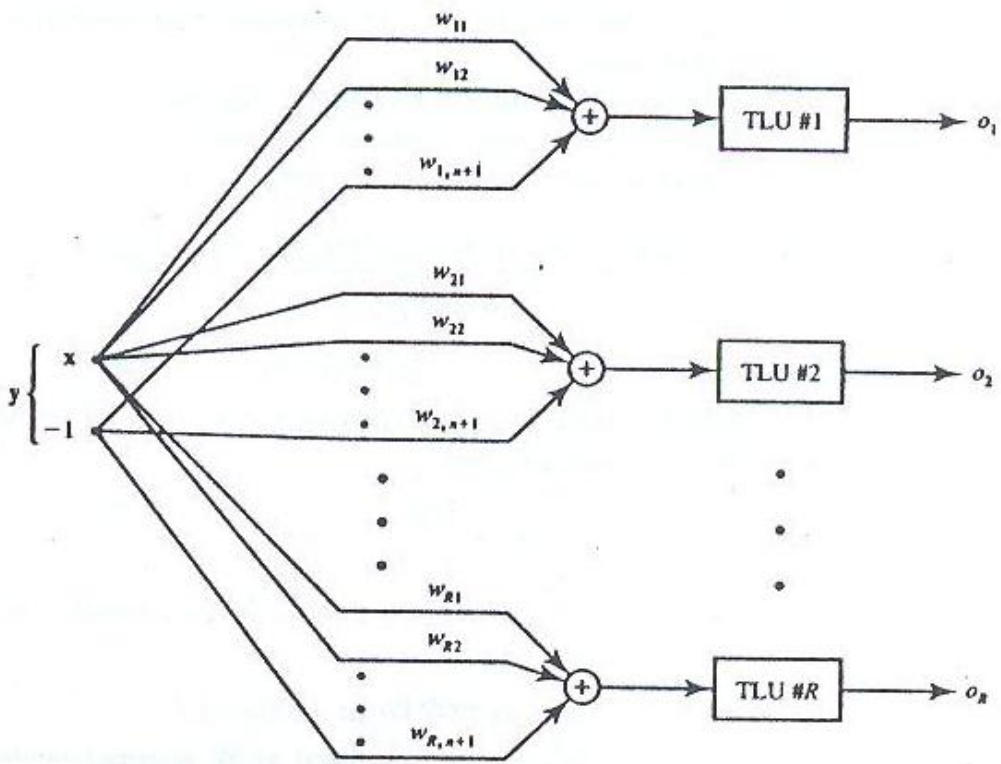
- Learning is finding optimum weights W^* s.t.

$$\begin{cases} W^{*T} y > 0 & \text{for} x \in \S_1 \\ W^{*T} y < 0 & \text{for} x \in \S_2 \end{cases}$$

- Training is terminated when there is no error in classification, $(w^* = w^n = w^{n+1})$.
- Assume after n steps learning is terminated.
- It can be shown that n is bounded
- i.e., after limited number of updating, the weights converge to their optimum values

Multi-category Single Layer Perceptron

- The perceptron learning rule so far was limited for two category classification
- We want to extend it for multigategory classification
- The weight of each neuron (TLU) is updated independent of other weights.
- The k's TLU reponses +1 and other TLU's -1 to indicate class k



R-category linear classifier using *R* discrete perceptrons.

R-Category Discrete Perceptron Training Algorithm

- Given P training pairs $\{x_1, d_1,\ x_2, d_2,\ ..., x_p, d_p\}$ where x_i is $(n \times 1),\ d_i$ is $(R \times 1),\ i = 1, ..., P$
- The augmented input vectors are $y_i = \begin{bmatrix} x_i \\ -1 \end{bmatrix}$, for $i = 1, ..., P$
- In the following, k is training step and p is step counter within training cycle.
  1. Choose $c > 0$
  2. Initialized weights at small random values, $W = [w_{ij}]$ is $R \times (n+1)$
  3. Initialize counters and error: $k \longleftarrow 1,\ p \longleftarrow 1, E \longleftarrow 0$
  4. Training cycle begins here. Set $y \longleftarrow y_p, d \longleftarrow d_p,\ o_i = sgn(w_i^T y)$ for $i = 1, ..., R$ (sgn is sign function)
  5. Update weights $w_i \longleftarrow w_i + \frac{1}{2}c(d_i - o_i)y$ for $i = 1, ..., R$
  6. Find error: $E \longleftarrow \frac{1}{2}(d_i - o_i)^2 + E$ for $i = 1, ..., R$
  7. If $p < P$ then $p \longleftarrow p + 1, k \longleftarrow k + 1$, go to step 4, otherwise, go to step 8.
  8. If $E = 0$ the training is terminated, otherwise $E \longleftarrow 0, p \longleftarrow 1$ go to step 4 for new training cycle

Example

- Revisit the three classes example
- The discriminant values are

Discriminant	Class 1 $[10\ 2]'$	Class 2 $[2\ -5]'$	Class 3 $[-5\ 5]'$
$g_1(x)$	52	-42	-92
$g_2(x)$	-4.5	14.5	-49.5
$g_3(x)$	-65	-60	25

- So the thresholds values: w_{13}, w_{23}, and w_{33} are 52, 14.5, and 25, respectively.
- Assume additional threshold $T_1 = T_2 = T_3 = -2$ so the threshold are changed to 50, 12.5, and 23, respectively.

- Now use perceptron learning rule:
- Consider randomly chosen initial values:
  $w_1^1 = [1 \ -2 \ 0]', \ w_2^1 = [0 \ -1 \ 2]', \ w_3^1 = [1 \ 3 \ -1]'$
- Use the patterns in sequence to update the weights:
  - y_1 is input:

$$sgn([1 \ -2 \ 0]\begin{bmatrix} 10 \\ 2 \\ -1 \end{bmatrix}) = 1$$

$$sgn([0 \ -1 \ 2]\begin{bmatrix} 10 \\ 2 \\ -1 \end{bmatrix}) = -1$$

$$sgn([1 \ 3 \ -1]\begin{bmatrix} 10 \\ 2 \\ -1 \end{bmatrix}) = 1^*$$

  - TLU # 3 has incorrect response. So
    $w_1^2 = w_1^1, \ w_2^2 = w_2^1, \ w_3^2 = [1 \ 3 \ -1]' - [10 \ 2 \ -1]' = [-9 \ 1 \ 0]'$

- y_2 is input:

$$sgn([1 \ -2 \ 0]\begin{bmatrix} 2 \\ -5 \\ -1 \end{bmatrix}) = 1^*$$

$$sgn([0 \ -1 \ 2]\begin{bmatrix} 2 \\ -5 \\ -1 \end{bmatrix}) = 1$$

$$sgn([-9 \ 1 \ 0]\begin{bmatrix} 2 \\ -5 \\ -1 \end{bmatrix}) = -1$$

  - TLU # 1 has incorrect response. So
    $w_1^3 = [1 \ 2 \ 0]' - [2 \ -5 \ -1]' = [-1 \ 3 \ 1]', \ w_2^3 = w_2^2, \ w_3^3 = w_3^2$

- y_3 is input:

$$sgn([-1\ 3\ 1]\begin{bmatrix}-5\\5\\-1\end{bmatrix}) = 1^*$$

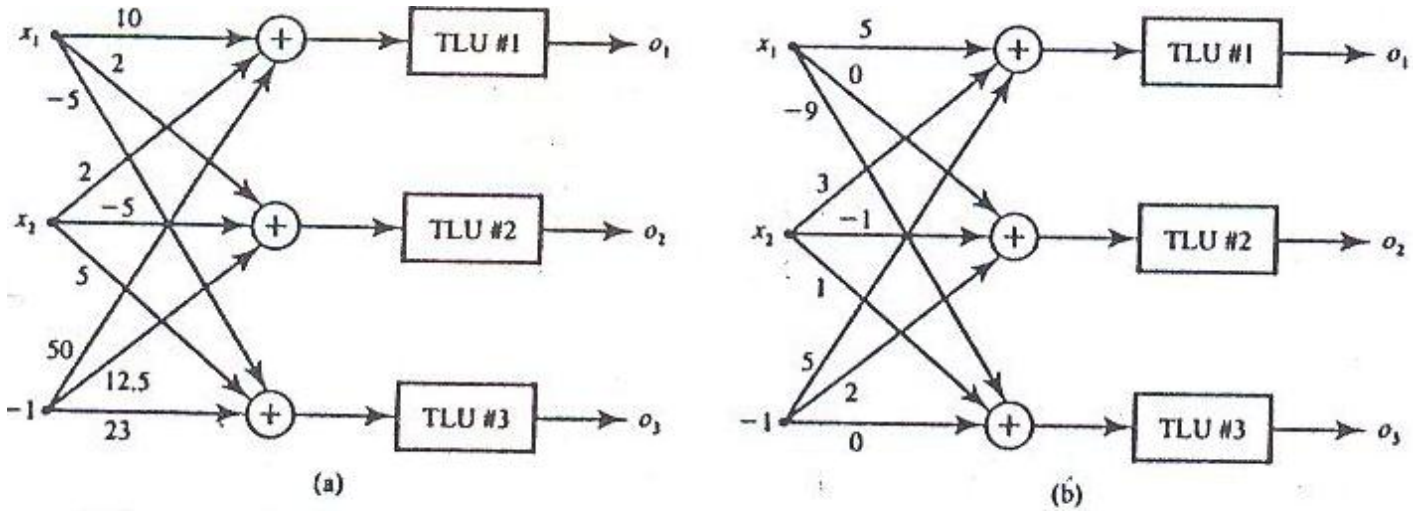
$$sgn([0\ -1\ 2]\begin{bmatrix}-5\\5\\-1\end{bmatrix}) = -1$$

$$sgn([-9\ 1\ 0]\begin{bmatrix}-5\\5\\-1\end{bmatrix}) = 1$$

  - TLU # 1 has incorrect response. So $w_1^4 = [4\ -2\ 2]'$, $w_2^4 = w_2^3$, $w_3^4 = w_3^3$

- First learning cycle is finished but the error is not zero, so the training is not terminated

- In next training cycles TLU # 2 and 3 are correct.
- TLU # 1 is changed as follows
  $w_1^5 = w_1^4,\ w_1^6 = [2\ 3\ 3]',\ w_1^7 = [7\ -2\ 4]',\ w_1^8 = w_1^7,\ w_1^9 = [5\ 3\ 5]$
- The trained network is
$$\begin{aligned} o_1 &= sgn(5x_1 + 3x_2 - 5) \\ o_2 &= sgn(-x_2 - 2) \\ o_1 &= sgn(-9x_1 + x_2) \end{aligned}$$
- The discriminant functions for classification are not unique.



(a) three-perceptron classifier from maxi- mum selector, (b) three-perceptron trained classifier,

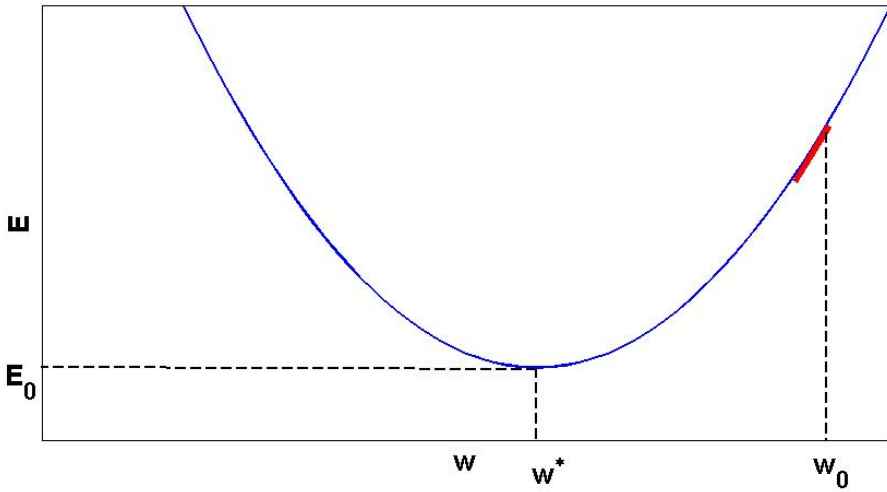
Continuous Perceptron

- In many cases the output is not necessarily limited to two values (± 1)
- Therefore, the activation function of NN should be continuous
- The training is indeed defined as adjusting the optimum values of the weights, s.t. minimize a criterion function
- This criterion function can be defined based on error between the network output and desired output.
- Sum of square root error is a popular error function
- The optimum weights are achieved using gradient or steepest decent procedure.

- Consider the error function: $E = E^0 + \lambda(w - w^*)^2 \rightsquigarrow \frac{dE}{dw} = 2\lambda(w - w^*), \frac{d^2E}{dw^2} = 2\lambda$
- The problem is finding w^* s.t min E
- To achieve min error at $w = w^*$ from initial weight w_0, the weights should move in direction of negative gradient of the curve.
- $\therefore$ The updating rule is

$$w^{k+1} = w^k - \eta \nabla E(w^k)$$

where η is pos. const. called learning constant.



- The error to be minimized is

$$E^k = \frac{1}{2}(d^k - o^k)^2$$
$$o^k = f(net^k)$$

- For simplicity superscript k is skipped. But remember the weights updates is doing at kth training step.
- The gradient vector is

$$\nabla E(w) = -(d-o)f'(net)\begin{bmatrix} \frac{\partial(net)}{\partial w_1} \\ \frac{\partial(net)}{\partial w_2} \\ \vdots \\ \frac{\partial(net)}{\partial w_{n+1}} \end{bmatrix}$$

- $net = w^T y \rightsquigarrow \frac{\partial(net)}{\partial w_i} = y_i$ for $i = 1, ..., n+1$
- $\therefore \nabla E = -(d-o)f'(net)y$

- The TLU activation function is not useful, since its time derivative is always zero and indefinite at $net = 0$
- Use sigmoid activation function

$$f(net) = \frac{2}{1 + exp(-net)} - 1$$

- Time derivative of sigmoid function can be expressed based on the function itself

$$f'(net) = \frac{2exp(-net)}{1 + exp(-net)} = \frac{1}{2}(1 - f(net)^2)$$

- $o = f(net)$, therefore,

$$\nabla E(w) = -\frac{1}{2}(d - o)(1 - o^2)y$$

- Finally the updating rule is

$$w^{k+1} = w^k + \frac{1}{2}\eta(d^k - o^k)(1 - o^{k2})y^k \tag{8}$$

- Comparing the updating rule of continuous perceprton (8) with the discrete perceptron learning ($w^{k+1} = w^k + \frac{c}{2}(d^k - o^k)y^k$)
  - The correction weights are in the same direction
  - Both involve adding/subtracting a fraction of the pattern vector y
  - The essential difference is scaling factor $1 - o^{k2}$ which is always positive and smaller than 1.
  - In continuous learning, a weakly committed perceptron (*net* close to zero) the correction scaling factor is larger than the more close responses with large magnitude.

Single Continuous Perceptron Training Algorithm

- Given P training pairs $\{x_1, d_1,\ x_2, d_2,\ ..., x_p, d_p\}$ where x_i is $(n \times 1),\ d_i$ is $(1 \times 1),\ i = 1, ..., P$
- The augmented input vectors are $y_i = [x_i\ \ -1]^T$, for $i = 1, ..., P$
- In the following, k is training step and p is step counter within training cycle.
  1. Choose $\eta > 0,\ \lambda = 1, E_{max} > 0$
  2. Initialized weights at small random values, w is $(n \times 1) \times 1$
  3. Initialize counters and error: $k \longleftarrow 1,\ p \longleftarrow 1, E \longleftarrow 0$
  4. Training cycle begins here. Set $y \longleftarrow y_p, d \longleftarrow d_p,\ o = f(w^T y)$ (f(net) is sigmoid function)
  5. Update weights $w \longleftarrow w + \frac{1}{2}\eta(d - o)(1 - o^2)y$
  6. Find error: $E \longleftarrow \frac{1}{2}(d - o)^2 + E$
  7. If $p < P$ then $p \longleftarrow p + 1, k \longleftarrow k + 1$, go to step 4, otherwise, go to step 8.
  8. If $E < E_{max}$ the training is terminated, otherwise $E \longleftarrow 0, p \longleftarrow 1$ go to step 4 for new training cycle.

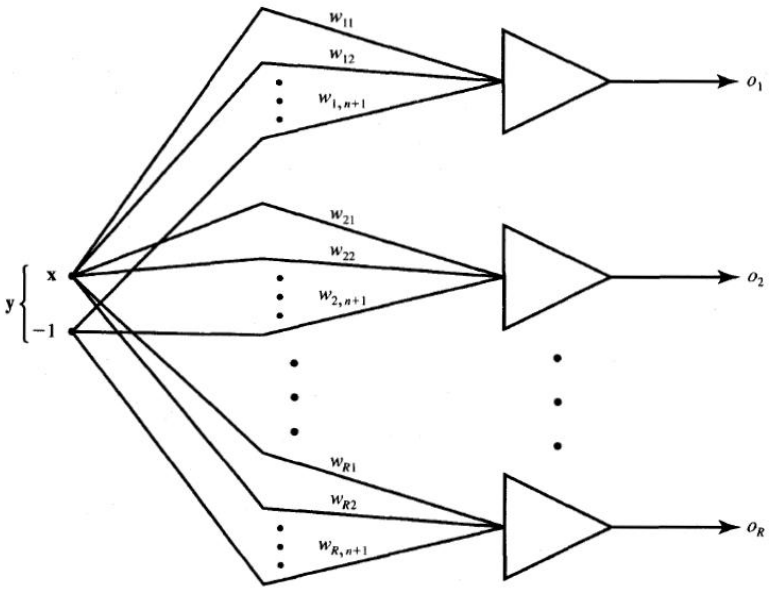
R-Category Continues Perceptron

- Gradient training rule derived for $R = 2$ is also applicable for multi-category classifier
- The training rule with be changed to

$$w_i^{k+1} = w_i^k + \frac{1}{2}\eta(d_i^k - o_i^k)(1 - o_i^{k2})y^k,$$
$$for\ i = 1, ..., R$$

- It is equivalent to individual weight adjustment

$$w_{ij}^{k+1} = w_{ij}^k + \frac{1}{2}\eta(d_i^k - o_i^k)(1 - o_i^{k2})y_j^k,$$
$$for\ j = 1, ..., n+1,\ \ i = 1, ..., R$$



R-category linear classifier using continuous perceptrons.