Computational Intelligence
Lecture 11: Fuzzy Control III

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Adaptive Fuzzy Control

- The objective of adaptive control: Providing desired performance in presence of uncertainties.
- The main advantage of adaptive fuzzy control comparing to classical adaptive control:
  - To obtain control adaptive law, the knowledge of experts on system dynamics and/or control strategies can be considered.
- Expert knowledge can be categorized to
  - System knowledge: The If-then rules which describe the unknown system behavior.
    - For example: For a car:"IF you push the gas pedal more, Then the car speed is increased.
  - Control knowledge: the rule of fuzzy control which indicates at each situation, which control action is required.
    - For example: For a car:"IF the speed is low, Then push the gas pedal more.

- Based on the applied type of expert knowledge, the adaptive fuzzy control can be
  - Indirect adaptive fuzzy control: The fuzzy control includes some fuzzy systems made based on system knowledge
  - Direct adaptive fuzzy control: The control fuzzy includes a fuzzy system which is made based on control knowledge
  - Combination of indirect/direct adaptive fuzzy control: A weighted combination of direct and indirect adaptive control
- **Indirect Adaptive Fuzzy Control**
  - Consider nth order nonlinear system

$$\begin{aligned} x^{(n)} &= f(x, \dot{x}, \ldots, x^{(n-1)}) + g(x, \dot{x}, \ldots, x^{(n-1)})u \\ y &= x \end{aligned}$$

    where $X = (x, \dot{x}, \ldots, x^{(n-1)})$: state vector; $u \in R$: input; $y \in R$: Output; f, g : unknown functions
  - Assume the system is controllable
  - Objective: find $u = u(X|\theta)$ based on fuzzy rules and an adaptation law for adjusting θ s.t. y tracks y_m

The Fuzzy Control Design for Indirect Adaptive Control

- Assume a set of IF-then laws based on system knowledge is available to describe the I/O behavior of g and f

$$\text{If } x_1 \text{ is } F_1^r, \ldots, x_n \text{ is } F_n^r, \text{then } f(x) \text{ is } C^r \tag{1}$$
$$\text{If } x_1 \text{ is } G_1^s, \ldots, x_n \text{ is } G_n^s, \text{then } g(x) \text{ is } D^r$$

$r = 1, 2, \ldots, L_f$ $s = 1, 2, \ldots, L_g$

- If the f and g functions are known, u is selected s.t. cancel the nonlinearities and control based on linear control techniques such as pole-placement:

$$u^* = \frac{1}{g(x)}[-f(x) + y_m^{(n)} + K^T e] \tag{2}$$

where $e = y_m - y$ is dynamics error, $K = (k_1, \ldots, k_n)^T$, s.t. the roots of $s^n + k_1 s^{n-1} + \ldots + k_n$ are LHP

- Since f and g are unknown, the estimation of them are considered in (3):

$$u^* = \frac{1}{\hat{g}(X|\theta_g)}[-\hat{f}(X|\theta_f) + y_m^{(n)} + K^T e] \tag{3}$$

- $\hat{g}(X|\theta_g)$ and $\hat{f}(X|\theta_f)$ are obtained in the following two steps
  1. for $x_i,\ i = 1, \dots, n$, define p_i fuzzy set of $A_i^{l_i}, l_i = 1, \dots, p_i$, s.t. they include $F_i^r,\ r = 1, \dots, L_f$ in(1); also define q_i fuzzy set of $B_i^{l_i}, l_i = 1, \dots, q_i$, s.t. they include $G_i^s,\ s = 1, \dots, L_g$ in(1)
  2. Using the fuzzy rule $\prod_{i=1}^{n} p_i$ provide a fuzzy system for $\hat{f}(X|\theta_f)$:

$$\text{If}\ \ x_1\ \ \text{is}\ \ A_1^{l_1}, \dots, x_n\ \ \text{is}\ \ A_n^{l_n}, \text{then}\ \ \hat{f}(x)\ \text{is}\ E^{l_1,\dots,l_n}, \tag{4}$$

for $l_i = 1, \dots, p_i, i = 1, \dots, n$

  - If the If part of (1) is the same as If part of (4), then $E^{l_1,\dots,l_n}$ is C^r.
  - Otherwise, it is considered as a new fuzzy set

- Using the fuzzy rule $\prod_{i=1}^{n} q_i$ provide fuzzy system for $\hat{g}(X|\theta_g)$:

$$\text{If}\ \ x_1\ \ \text{is}\ \ B_1^{l_1}, \dots, x_n\ \ \text{is}\ \ B_n^{l_n}, \text{then}\ \ \hat{g}(x)\ \text{is}\ H^{l_1,\dots,l_n} \tag{5}$$

for $l_i = 1, \dots, q_i, i = 1, \dots, n$

- If the If part of (1) is the same as If part of (5), then $H^{l_1,\dots,l_n}$ is D^r.
- Otherwise, it is considered as a new fuzzy set

- Consider:
  Inference engine: production; Fuzzifier: singleton; Difizzifier: center average

$$f(X|\theta_f) = \frac{\sum_{l_1=1}^{p_1} \cdots \sum_{l_n=1}^{p_n} \bar{y}_f^{l_1 \ldots l_n} [\prod_{i=1}^{n} \mu_{A_i}{}^{l_i}(x_i)]}{\sum_{l_1=1}^{p_1} \cdots \sum_{l_n=1}^{p_n} [\prod_{i=1}^{n} \mu_{A_i}{}^{l_i}(x_i)]} \quad (6)$$

$$g(X|\theta_g) = \frac{\sum_{l_1=1}^{q_1} \cdots \sum_{l_n=1}^{q_n} \bar{y}_g^{l_1 \ldots l_n} [\prod_{i=1}^{n} \mu_{B_i}{}^{l_i}(x_i)]}{\sum_{l_1=1}^{q_1} \cdots \sum_{l_n=1}^{q_n} [\prod_{i=1}^{n} \mu_{B_i}{}^{l_i}(x_i)]} \quad (7)$$

- Consider $\bar{y}_f^{l_1 \ldots l_n}$ and $\bar{y}_g^{l_1 \ldots l_n}$ are free parameters which are summed in $\theta_f \in R^{\prod_{i=1}^{n} p_i}$ and $\theta_g \in R^{\prod_{i=1}^{n} q_i}$, respectively:

$$f(X|\theta_f) = \theta_f^T \varepsilon(X)$$

$$g(X|\theta_g) = \theta_g^T \eta(X)$$

$$\varepsilon(X) = \frac{\prod_{i=1}^{n} \mu_{A_i}{}^{l_i}(x_i)}{\sum_{l_1=1}^{p_1} \cdots \sum_{l_n=1}^{p_n} [\prod_{i=1}^{n} \mu_{A_i}{}^{l_i}(x_i)]} \quad (8)$$

$$\eta(X) = \frac{\prod_{i=1}^{n} \mu_{B_i}{}^{l_i}(x_i)}{\sum_{l_1=1}^{q_1} \cdots \sum_{l_n=1}^{q_n} [\prod_{i=1}^{n} \mu_{B_i}{}^{l_i}(x_i)]} \quad (9)$$

- Adapting Rule:

$$\begin{aligned}\dot{\theta}_f &= -\gamma_1 e^T Pb\varepsilon(X)\\ \dot{\theta}_g &= -\gamma_2 e^T Pb\eta(X)\end{aligned} \quad (10)$$

- where $-\gamma_1$, $-\gamma_2$ are pos. numbers and P is Pos. def. matrix obtained from Lyapunov equation $\Lambda^T P + P\Lambda = -Q, Q > 0$

$$\Lambda = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & . \\ . & . & . & \dots & . \\ . & . & . & \dots & . \\ 0 & 0 & 0 & \dots & 1 \\ -k_n & -k_{n-1} & \dots & \dots & -k_1 \end{bmatrix}, b = \begin{bmatrix} 0 \\ 0 \\ . \\ . \\ 0 \\ 1 \end{bmatrix}$$

- It should be mentioned that the system knowledge (1) is considered on selecting $\theta_f(0), \theta_g(0)$

Indirect Adaptive Fuzzy Control

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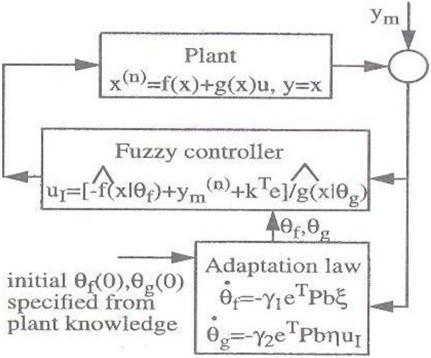

Direct Adaptive Fuzzy Control

- Consider nth order nonlinear system

$$\begin{aligned} x^{(n)} &= f(x, \dot{x}, \ldots, x^{(n-1)}) + bu \\ y &= x \end{aligned}$$

  where $X = (x, \dot{x}, \ldots, x^{(n-1)})$: state vector; $u \in R$: input; $y \in R$: Output; f : unknown functions, $b > 0$ is cons. and unknown
- Assume the system is controllable
- Objective: find $u = u(X|\theta)$ based on fuzzy rules and an adaptation law for adjusting θ s.t. y tracks y_m
- Main difference of direct and indirect adaptive fuzzy control is type of available expert knowledge
- In direct adaptive fuzzy control, assume a set of IF-then laws based on *control knowledge*

$$\text{If } \; x_1 \; \text{ is } \; P_1^r, \ldots, x_n \; \text{ is } \; P_n^r, \text{then } \; u \text{ is } Q^r, \quad r = 1, 2, \ldots, L_u \tag{11}$$

The Fuzzy Control Design

- $u_D(X|\theta_f)$ is obtained in the following two steps
  1. For x_i, $i = 1, \ldots, n$, define m_i fuzzy set of $A_i^{l_i}, l_i = 1, \ldots, m_i$, s.t. they include p_i^r, $r = 1, \ldots, L_u$ in(11)
  2. Using the fuzzy rule $\prod_{i=1}^{n} m_i$ provide fuzzy system for $u(X|\theta_u)$:

$$\text{If } \; x_1 \; \text{ is } \; A_1^{l_1}, \ldots, x_n \; \text{ is } \; A_n^{l_n}, \text{then } \; u \text{ is } S^{l_1,\ldots,l_n}, \tag{12}$$

  for $l_i = 1, \ldots, m_i, i = 1, \ldots, n$
    - If the If part of (12) is the same as If part of (11), then $S^{l_1,\ldots,l_n}$ is Q^r.
    - Otherwise, it is considered as a new fuzzy set
- Consider: Inference engine: Production; Fuzzifier: singleton; Difizzifier: center mean

$$u(X|\theta_f) = \frac{\sum_{l_1=1}^{m_1} \cdots \sum_{l_n=1}^{m_n} \bar{y}_u^{l_1 \ldots l_n} [\prod_{i=1}^{n} \mu_{A_i}{}^{l_i}(x_i)]}{\sum_{l_1=1}^{m_1} \cdots \sum_{l_n=1}^{m_n} [\prod_{i=1}^{n} \mu_{A_i}{}^{l_i}(x_i)]} \tag{13}$$

- Consider $\bar{y}_u^{l_1 \cdots l_n}$ are adjustable parameters, summed in $\theta_u \in R^{\prod_{i=1}^{n} p_i}$:

$$\begin{aligned} u(X|\theta_u) &= \theta_u^T \varepsilon(X) \\ \varepsilon(X) &= \frac{\prod_{i=1}^{n} {\mu_{A_i}}^{l_i}(x_i)}{\sum_{l_1=1}^{m_1} \cdots \sum_{l_n=1}^{m_n} [\prod_{i=1}^{n} {\mu_{A_i}}^{l_i}(x_i)]} \end{aligned} \quad (14)$$

- Adapting Rule:

$$\dot{\theta}_u = \gamma_3 e^T p_n \varepsilon(X)$$

- where $-\gamma_3$, is pos. numbers and p_n is the last column of P is Pos. def. matrix obtained from Lyapunov equation

$$\Lambda^T P + P\Lambda = -Q, Q > 0, \quad \Lambda = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & . \\ . & . & . & \dots & . \\ . & . & . & \dots & . \\ 0 & 0 & 0 & \dots & 1 \\ -k_n & -k_{n-1} & \dots & \dots & -k_1 \end{bmatrix}$$

Direct Adaptive Fuzzy Control

- Direct Adaptive Fuzzy Control