

Nonlinear Control

Lecture 1: Introduction

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Motivations

- ▶ A system is called **linear** if its behavior set satisfies linear superposition laws: .i.e. $\forall z_1, z_2 \in \mathcal{B}$ and constant $c \in \mathcal{R} \rightsquigarrow z_1 + z_2 \in \mathcal{B}, cz_1 \in \mathcal{B}$
- ▶ A **nonlinear system** is simply a system which is not linear.
- ▶ Powerful tools founded based on superposition principle make analyzing the linear systems simple.
- ▶ All practical systems posses nonlinear dynamics.
- ▶ Sometimes it is possible to describe the operation of physical systems by linear model around its operating points
- ▶ Linearized system can provide us an approximate behavior of the nonlinear system
- ▶ **But** in analyzing the overall system behavior, often linearized model *inadequate or inaccurate*.

- ▶ Linearization is an approximation in the neighborhood of an operating system $\rightsquigarrow$ it can only predict **local** behavior of nonlinear system. (No info regarding *nonlocal* or *global* behavior of system)
- ▶ Due to richer dynamics of nonlinear systems comparing to the linear ones, there are some essentially nonlinear phenomena that can take place only in presence of nonlinearity
- ▶ **Essentially nonlinear phenomena**
 - ▶ **Finite escape time:** The state of linear system goes to infinity as $t \rightarrow \infty$; nonlinear system's state can go to infinity in *finite time*.
 - ▶ **Multiple isolated equilibria:** linear system can have only one isolated equilibrium point which attracts the states irrespective of the initial state; nonlinear system can have more than one isolated equilibrium point, the state may converge to each depending on the initial states.
 - ▶ **Limit cycle:** There is no robust oscillation in linear systems. To oscillate there should be a pair of eigenvalues on the imaginary axis which due to presence of perturbations it is almost impossible in practice; For nonlinear systems, there are some oscillations named limit cycle with fixed amplitude and frequency.

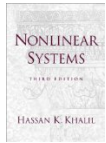
Essentially nonlinear phenomena

- ▶ **Subharmonic,harmonic or almost periodic oscillations:** A stable linear system under a periodic input $\rightsquigarrow$ output with the same frequency; A nonlinear system under a periodic input $\rightsquigarrow$ can oscillate with submultiple or multiple frequency of input or almost-periodic oscillation.
- ▶ **Chaos:** A nonlinear system may have a different steady-state behavior which is not equilibrium point, periodic oscillation or almost-periodic oscillation. This chaotic motions exhibit random, despite of deterministic nature of the system.
- ▶ **Multiple modes of behavior:** A nonlinear system may exhibit multiple modes of behavior based on type of excitation:
 - ▶ an unforced system may have one limit cycle.
 - ▶ Periodic excitation may exhibit harmonic, subharmonic,or chaotic behavior based on amplitude and frequency of input.
 - ▶ if amplitude or frequency is smoothly changed, it may exhibit discontinuous jump of the modes as well.

- ▶ **Linear systems**: can be described by a set of ordinary differential equations and usually the *closed-form expressions* for their solutions are derivable. **Nonlinear systems**: In general this is not possible $\rightsquigarrow$ It is desired to make a prediction of system behavior even in absence of closed-form solution. this type of analysis is called **qualitative analysis**.
- ▶ Despite of **linear systems**, no tool or methodology in **nonlinear system** analysis is *universally* applicable $\rightsquigarrow$ their analysis requires a wide verity of tools and higher level of mathematic knowledge
- ▶ $\therefore$ stability analysis and stabilizability of such systems and getting familiar with associated control techniques is the basic requirement of graduate studies in control engineering.
- ▶ **The aim of this course are**
 - ▶ developing a basic understanding of nonlinear control system theory and its applications.
 - ▶ introducing tools such as Lyapunov's method analyze the system stability
 - ▶ Presenting techniques such as feedback linearization to control nonlinear systems.

Reference Books

- ▶ **Text Book:** Nonlinear Systems, H. K. Khalil, 3rd edition, Prentice-Hall, 2002
- ▶ **Other reference Books:**
- ▶ Applied Nonlinear Control, J. J. E. Slotine, and W. Li, Prentice-Hall, 1991
- ▶ Nonlinear System Analysis, M. Vidyasagar, 2nd edition, Prentice-Hall, 1993
- ▶ Nonlinear Control Systems, A. Isidori, 3rd edition Springer-Verlag, 1995



Topics

Topic	Date	Refs
Introduction	Week 1	
Phase plane Analysis and Fundamental Properties	Week 2,3	Chapter 1-3
Stability Theory	Week 4-7	Chapter 4,8,9
Input-to-state, I/O stability and Absolute stability	Week 8,9	Chapter 4,5
Passivity	Week 10	Chapter 10
Feedback linearization	Week 11	Chapter 12,13
Some nonlinear design tools	Week 12,13	Chapter 14

- At this course we consider dynamical systems modeled by a finite number of coupled first-order ordinary differential equations:

$$\dot{x} = f(t, x, u) \quad (1)$$

where $x = [x_1, \dots, x_n]^T$: state vector, $u = [u_1, \dots, u_p]^T$: input vector, and $f(.) = [f_1(.), \dots, f_n(.)]^T$: a vector of nonlinear functions.

- Eq. (1) is called *state equation*.
- Another equation named *output equation*:

$$y = h(t, x, u) \quad (2)$$

where $y = [y_1, \dots, y_q]^T$: output vector.

- Equ (2) is employed for particular interest in analysis such as
 - variables which can be measured physically
 - variables which are required to behave in a desirable manner
- Eqs (1) and (2) together are called **state-space model**.

- Most of our analysis are dealing with **unforced state equations** where u does not present explicitly in Equ (1):

$$\dot{x} = f(t, x)$$

- In unforced state equations, input to the system is **NOT** necessarily zero.
- Input can be a function of time: $u = \gamma(t)$, a feedback function of state: $u = \gamma(x)$, or both $u = \gamma(t, x)$ where is substituted i Equ (1).
- **Autonomous** or **Time-invariant Systems**:

$$\dot{x} = f(x) \quad (3)$$

- function of f does not explicitly depend on t .
- Autonomous systems are invariant to shift in time origin, i.e. changing t to $\tau = t - a$ does not change f .
- The system which is not autonomous is called **nonautonomous** or **time-varying**.

► **Equilibrium Point** $x = x^*$

- x^* in state space is equilibrium point if whenever the state starts at x^* , it will remain at x^* for all future time.
- for autonomous systems (3), the equilibrium points are the real roots of equation: $f(x) = 0$.
- Equilibrium point can be
 - **Isolated**: There are no other equilibrium points in its vicinity.
 - **a continuum of equilibrium points**

Pendulum

- Employing Newton's second law of motion, equation of pendulum motion is:

$$ml\ddot{\theta} = -mg \sin \theta - kl\dot{\theta}$$

l : length of pendulum rod;

m : mass of pendulum bob;

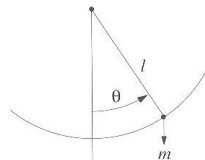
k : coefficient of friction;

θ : angle subtended by rod and vertical axis

- To obtain state space model,
let $x_1 = \theta$, $x_2 = \dot{\theta}$:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\frac{g}{l} \sin x_1 - \frac{k}{m} x_2$$



Pendulum.

Pendulum

- ▶ To find equilibrium point: $\dot{x}_1 = \dot{x}_2 = 0$

$$0 = x_2$$

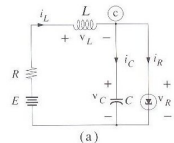
$$0 = -\frac{g}{l}\sin x_1 - \frac{k}{m}x_2$$

- ▶ The Equilibrium points are at $(n\pi, 0)$ for $n = 0, \pm 1, \pm 2, \dots$
 - ▶ Pendulum has two equilibrium points: $(0, 0)$ and $(\pi, 0)$,
 - ▶ Other equilibrium points are repetitions of these two which correspond to number of pendulum full swings before it rests
- ▶ Physically we can see that the pendulum rests at $(0, 0)$, but hardly maintain rest at $(\pi, 0)$

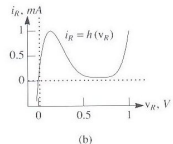
Tunnel Diode Circuit

- ▶ The tunnel diode is characterized by $i_R = h(v_R)$
- ▶ The energy-storing elements are C and L which assumed are linear and time-invariant $i_C = C \frac{dv_C}{dt}$, $v_L = L \frac{di_L}{dt}$.
- ▶ Employing Kirchhoff's current law:
 $i_C + i_R - i_L = 0$
- ▶ Employing Kirchhoff's voltage law:
 $v_C - E + Ri_L + v_L = 0$
- ▶ for state=space model, let $x_1 = v_C$, $x_2 = i$
 $u = E$ as a constant input:

$$\begin{aligned}\dot{x}_1 &= \frac{1}{C}[-h(x_1) + x_2] \\ \dot{x}_2 &= \frac{1}{L}[-x_1 - Rx_2 + u]\end{aligned}$$



(a) Tunnel diode circuit



(b) tunnel diode $v_R - i_R$ characteristic.

Tunnel Diode Circuit

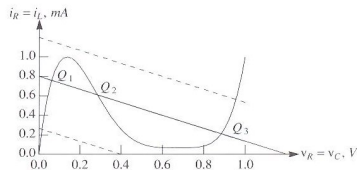
- To find equilibrium point: $\dot{x}_1 = \dot{x}_2 = 0$

$$0 = \frac{1}{C}[-h(x_1) + x_2]$$

$$0 = \frac{1}{L}[-x_1 - Rx_2 + u]$$

- Equilibrium points depends on E and R

$$x_2 = h(x_1) = \frac{E}{R} - \frac{1}{R}x_1$$



Equilibrium points of the tunnel diode circuit.

- For certain E and R , it may have 3 points (Q_1, Q_2, Q_3).
- if $E \uparrow$, and same $R \rightsquigarrow$ only Q_3 exists.
- if $E \downarrow$, and same $R \rightsquigarrow$ only Q_1 exists.