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Coupled magnetic equivalent circuits and the analytical solution in the air-gap of squirrel cage induction machines

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Abstract. In this paper a new method is presented to couple an analytical magnetic field solution of the air-gap of an induction machine with magnetic equivalent circuits method used to model the rest of the machine. The proposed method eliminates the need for complex static and dynamic mesh generation process in the air-gap. Therefore, the modelling of rotor motion becomes simpler and more accurate.

Keywords: Magnetic equivalent circuit, analytical method, squirrel cage induction machines

1. Introduction

The Magnetic Equivalent Circuit (MEC) method is one of the oldest methods to model magnetic devices. It is conventionally based on Ohm's and Kirchoff's laws for magnetic and electric circuits. Several works in the literature have been presented about the application of Magnetic Equivalent Circuits method to model induction machines [1–3]. However, one encounters some difficulties in modelling the air-gap of an induction machine. First, the air-gap is usually very thin and thus, it is difficult to fit a sufficient number of elements into it to model the important field interactions occurring in there. Next, the constructed MEC elements in the air-gap change as the rotor turns. Hence, re-meshing of the air-gap is required which makes the problem complicated [3].

To overcome the difficulties, Abdel-Razek et al. [4] and Feliachi et al. [5] have introduced the air-gap or macro element concept to model the air-gap of a rotating machine using the standard FEM.

This paper presents a new method by combining MEC with the analytical solution. The basic idea is similar to macro element method except that the interface conditions are different because of the use of scalar potentials in MEC method. The comparison of the proposed MEC simulation results with FEM proves the accuracy of the modified method.

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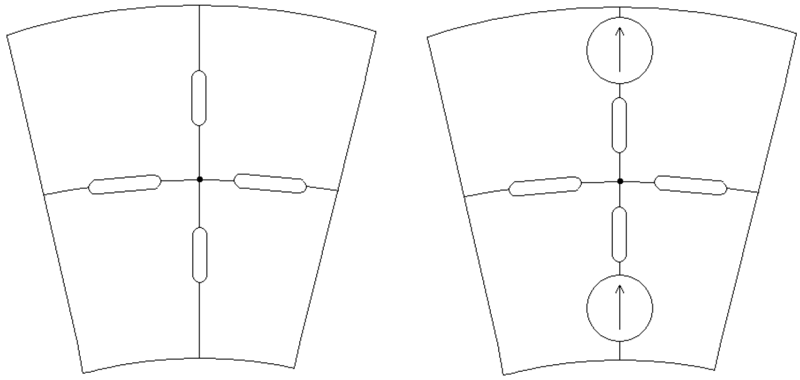

(a) (b)

Fig. 1. Equivalent Reluctance Model: (a) For non-conducting elements. (b) For conducting elements.

2. The proposed method

In this model, the scalar potential is used in MEC modelling while the analytical solution employs the magnetic vector potential.

2.1. *Magnetic equivalent circuits method*

To analyze the electromagnetic devices with magnetic equivalent circuit method, the related equations are as follows [3]:

$$\nabla \cdot \boldsymbol{B} = 0, \quad \nabla \times \boldsymbol{B} = \mu \boldsymbol{J}, \quad \nabla \times \boldsymbol{J} = -\sigma \frac{\boldsymbol{dB}}{\boldsymbol{dt}} = -\sigma \left(\frac{\partial B}{\partial t} + \omega_r \frac{\partial B}{\partial \theta} \right) \tag{1}$$

where, B, J, ω_r, σ, and μ are magnetic flux density, current density, rotational velocity of rotor, electrical conductivity, and magnetic permeability respectively.

For non-conducting elements, an equivalent reluctance model shown in Fig. 1a is used. Also, for conducting elements two additional MMF sources in radial orientation are used as shown in Fig. 1b.

In cylindrical coordinates, Eq. (1) can be rewritten as in the following:

$$\frac{\partial J_z}{\partial r} = \sigma \left[\frac{\partial B_\theta}{\partial t} + \omega_r \frac{\partial B_\theta}{\partial \theta} \right] \tag{2}$$

where, σ is the conductivity of rotor bars. The first term of the above equation denotes the induced current due to time variation caused by motor input voltage frequency and the second term denotes the induced current due to the relative motion between rotor and stator. The effect of relative motion between rotor and stator can be modeled by multiplying the conductivity by the slip of the rotor. Hence, Eq. (2) can be rewritten as below:

$$\frac{\partial J_z}{\partial r} \cong s\sigma \frac{\partial B_\theta}{\partial t} \tag{3}$$

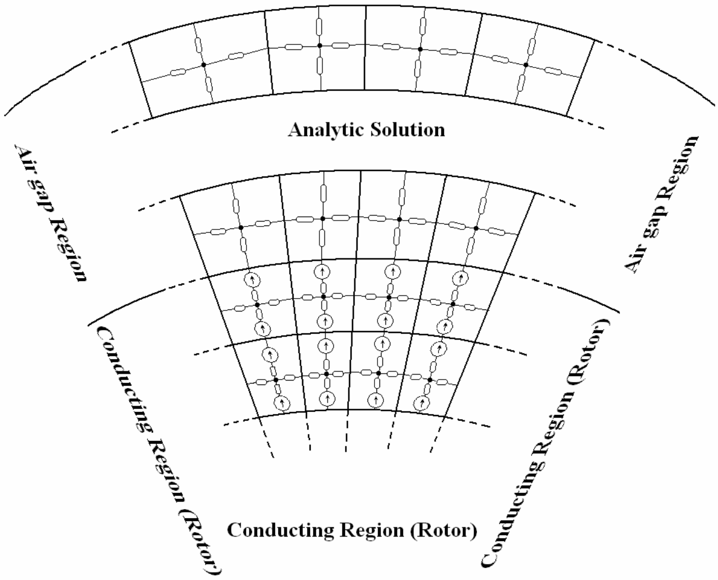

Fig. 2. Interface between MEC and analytical solution.

where, s is the slip of the rotor. Now, the induced current density in cylindrical coordinates can be written as in the following:

$$J_z = j\omega s\sigma \int_r B_\theta dr \tag{4}$$

where, ω is the source angular frequency. The induced current of element (i,k) due to time varying $(\Delta I_{z\omega}\,(i,k))$ can be obtained as follows:

$$\Delta I_z\,(i,k) = j\omega s\sigma \sum_{m=1}^{i} \left(\frac{U_{m,k+1} - U_{m,k}}{\left(R_{\theta(m,k+1)} + R_{\theta(m,k)}\right)\ L} \right) r\Delta r\Delta\theta \tag{5}$$

where, Δr and $\Delta\theta$ are the element's dimensions in radial and circumferential directions respectively, and L denotes the axial length of an induction machine. Also, $U_{i,k}$ and $R_{r(i,k)}$ are the scalar potential and the radial reluctance component of the node (i,k), respectively.

2.2. Analytical solution in the air-gap

In cylindrical coordinates, with vector potential A, Laplace's equation for an annular region is as bellow [4,5]:

$$\nabla^2 A = \frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\frac{\partial A}{\partial r}\right)\right] + \frac{1}{r^2}\frac{\partial^2 A}{\partial\theta^2} = 0 \tag{6}$$

Solutions take the form $A_n\,(r) = C_{1n}r^{-n-1} + C_{2n}r^{n-1}$ for the nth component.

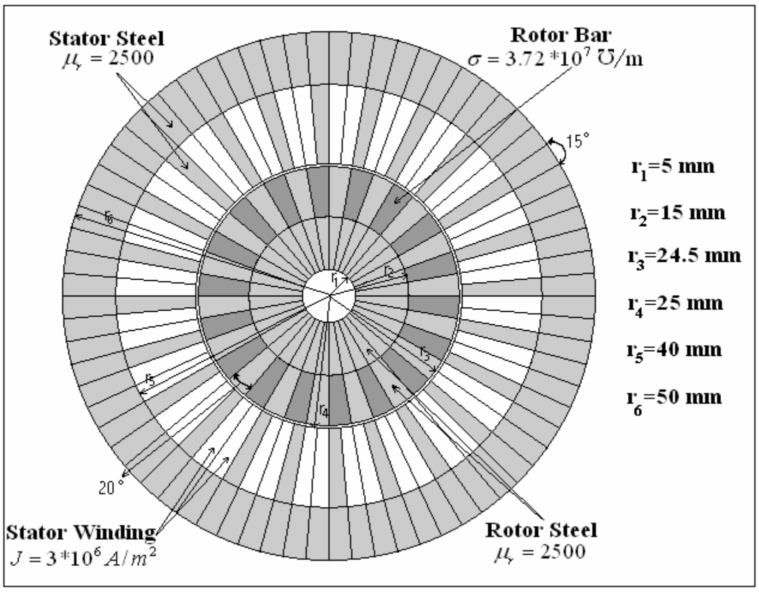

Fig. 3. The squirrel cage induction motor.

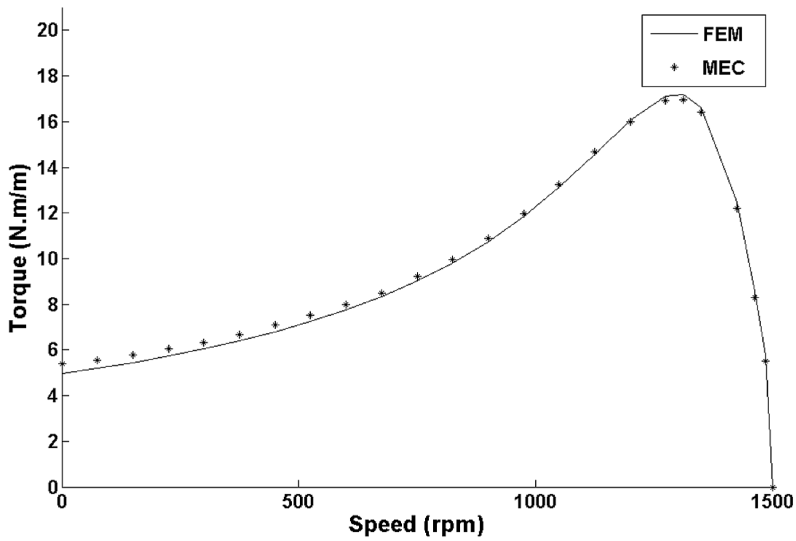

Fig. 4. Torque versus speed.

2.3. Coupling the two methods

The common boundaries of the two methods are two circles covering the air-gap. Boundary conditions between the two solutions are as in the following:

$$B_r\,(Analytical) = B_r\,(MEC)$$
$$H_\theta\,(Analytical) = H_\theta\,(MEC) \qquad (7)$$

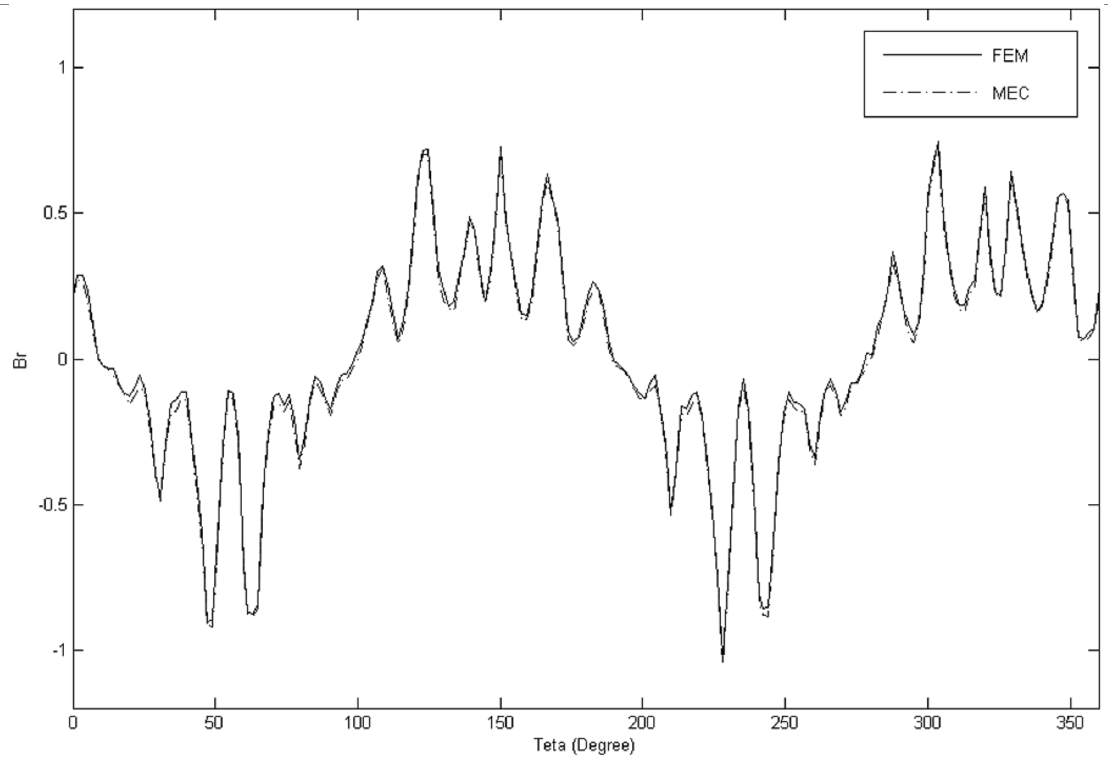

Fig. 5. Comparison of B_r in middle of air-gap at $s = 0.05$.

Figure 2 shows the interface between MEC and Analytical methods where the above boundary conditions are used to couple the two methods.

The scalar potentials of nodes in MEC modelling, and constants of the analytical solution are the variables that should be obtained.

3. Simulation results

The considered induction motor is shown in Fig. 3. For simplicity of MEC modelling, the slots and teeth in stator and rotor are shown as sectors of a circle. The torque versus speed calculated from the proposed method is shown and compared with FEM results in Fig. 4. Figures 5 and 6 show the radial and circumferential components of magnetic flux density along with the FEM results in the middle of air-gap, respectively. One can deduce from Figs 4 to 6 that the results of the proposed method compare very accurately with FEM.

4. Conclusion

This paper has presented a new method to couple an analytical magnetic field solution with the magnetic equivalent circuits method. The proposed method eliminates the need for complex dynamic mesh generation process in the air-gap. Simulation results prove the high accuracy of the proposed method.

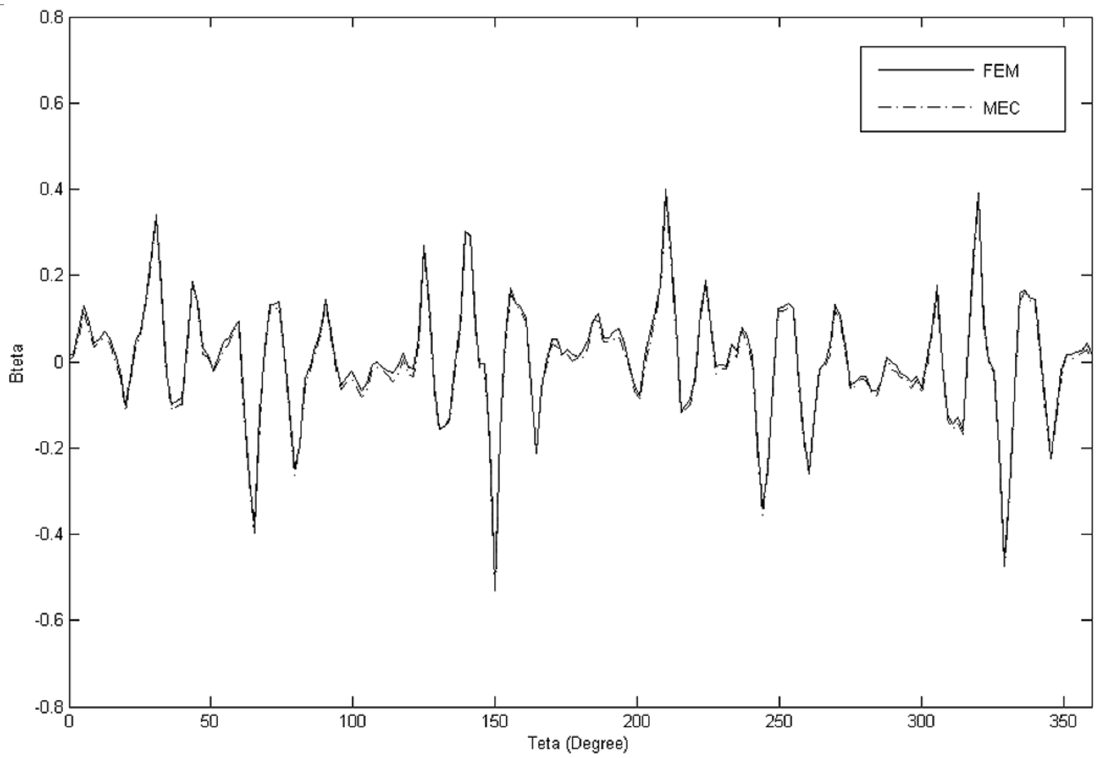

Fig. 6. Comparison of B_θ in middle of air-gap at $s = 0.05$.

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